Vectors in 3-D

Starter

1. **(Review of last lesson)** A boat is modelled as a particle moving across a horizontal surface with acceleration $\mathbf{a} = (e^{0.2t}\mathbf{i} + \sin t\,\mathbf{j})$ m/s² at time t seconds for $0 \le t \le 5$. At time $t = 0$, the boat is at $2\mathbf{j}$ m from the origin, moving in the $\mathbf{j}$ direction at a speed of 10 m/s. Find an expression for the boat's position vector, $\mathbf{s}$, at time t.

Working: $\mathbf{v} = \int (e^{0.2t}\mathbf{i} + \sin t\,\mathbf{j})dt = 5e^{0.2t}\mathbf{i} - \cos t\,\mathbf{j} + \mathbf{c}$

When $t = 0$, $\mathbf{v} = 10\mathbf{j} \Rightarrow 10\mathbf{j} = 5\mathbf{i} - \mathbf{j} + \mathbf{c} \Rightarrow \mathbf{c} = -5\mathbf{i} + 11\mathbf{j}$

$\mathbf{v} = (5e^{0.2t} - 5)\mathbf{i} + (11 - \cos t)\mathbf{j}$

$\mathbf{s} = \int (5e^{0.2t} - 5)\mathbf{i} + (11 - \cos t)\mathbf{j}dt$

$\mathbf{s} = (25e^{0.2t} - 5t)\mathbf{i} + (11t - \sin t)\mathbf{j} + \mathbf{p}$

When $t = 0$, $\mathbf{s} = 2\mathbf{j} \Rightarrow 2\mathbf{j} = 25\mathbf{i} + \mathbf{p} \Rightarrow \mathbf{p} = -25\mathbf{i} + 2\mathbf{j}$

$\therefore \mathbf{s} = (25e^{0.2t} - 5t - 25)\mathbf{i} + (2 + 11t - \sin t)\mathbf{j}$

So $\mathbf{s} = 5(5e^{0.2t} - t - 5)\mathbf{i} + (2 + 11t - \sin t)\mathbf{j}$

2. **(Review of AS material)** Let $\overrightarrow{OA} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} -7 \\ -8 \end{pmatrix}$. Find the vector $\overrightarrow{AB}$ and hence find the distance between A and B.

Working: $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} -7 \\ -8 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -11 \\ -7 \end{pmatrix}$

Distance $= \sqrt{(-11)^2 + (-7)^2} = \sqrt{170} = 13.0$ (3 s.f.)

E.g. 1 The triangle JKL has vertices at points J(4, 0, −3), K(−1, 3, 0) and L(2, 2, 7). Find the vectors $\overrightarrow{JK}$, $\overrightarrow{KL}$ and $\overrightarrow{LJ}$.

Working: $\overrightarrow{JK} = \mathbf{k} - \mathbf{j} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ 3 \end{pmatrix}$

Similarly $\overrightarrow{KL} = \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}$ and $\overrightarrow{LJ} = \begin{pmatrix} 2 \\ -2 \\ -10 \end{pmatrix}$

E.g. 2 Given that $\mathbf{a} = \mathbf{i} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{j} - \mathbf{k}$, find the values of $\mu \in \mathbb{R}$ such that $\mathbf{a} - \mu\mathbf{b}$ has magnitude $3\sqrt{2}$.

Working: $\mathbf{a} - \mu\mathbf{b} = \mathbf{i} - 3\mathbf{k} - \mu(2\mathbf{j} - \mathbf{k}) = \mathbf{i} - 2\mu\mathbf{j} + (\mu - 3)\mathbf{k}$

$|\mathbf{a} - \mu\mathbf{b}| = \sqrt{1^2 + (-2\mu)^2 + (\mu - 3)^2} = 3\sqrt{2}$

Squaring both sides and expanding: $1 + 4\mu^2 + \mu^2 - 6\mu + 9 = 18$

$5\mu^2 - 6\mu - 8 = 0 \Rightarrow (5\mu + 4)(\mu - 2) = 0$

$\therefore \mu = 2$ or $\mu = -0.8$

E.g. 3 Show that the points A, B and C are collinear if $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\overrightarrow{OB} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OC} = -\mathbf{i} + 8\mathbf{j} + 7\mathbf{k}$.

Working:

$\overrightarrow{AB} = -\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{BC} = -2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$

$\overrightarrow{BC} = -2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k} = 2 \times (-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) = 2 \times \overrightarrow{AB}$

Since $\overrightarrow{AB}$ is a multiple of $\overrightarrow{BC}$, the vectors must be parallel.

Since $\overrightarrow{AB}$ and $\overrightarrow{BC}$ have B as a common point and are parallel, it means that A, B and C are collinear.

Video A: Vectors in 3-D
Video B: Vectors in 3-D

Solutions to Starter and E.g.s

Exercise
p442 19D Qu 1i, 2i, 3i, 4ac, 5i, 6ac, 7ac, 8-12