Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 1

Monday 11 May 2020   Afternoon   Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

Answer **all** questions in the spaces provided.

1 Express the complex number $1 - \mathrm{i}\sqrt{3}$ in modulus-argument form.

Tick (✓) **one** box.

[1 mark]

$2\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$ ☐

$2\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)$ ☐

$2\left(\cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$ ☐

$2\left(\cos\left(-\frac{2\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{2\pi}{3}\right)\right)$ ☐

2 Given that $1 - \mathrm{i}$ is a root of the equation $z^3 - 3z^2 + 4z - 2 = 0$, find the other two roots.

Tick (✓) **one** box.

[1 mark]

$-1 + \mathrm{i}$ and -1 ☐

$1 + \mathrm{i}$ and 1 ☐

$-1 + \mathrm{i}$ and 1 ☐

$1 + \mathrm{i}$ and -1 ☐

Do not write outside the box

3 Given $(x-1)(x-2)(x-a) < 0$ and $a > 2$

Find the set of possible values of x.

Tick (✓) **one** box.

[1 mark]

$\{x : x < 1\} \cup \{x : 2 < x < a\}$ ☐

$\{x : 1 < x < 2\} \cup \{x : x > a\}$ ☐

$\{x : x < -a\} \cup \{x : -2 < x < -1\}$ ☐

$\{x : -a < x < -2\} \cup \{x : x > -1\}$ ☐

Turn over for the next question

Do not write outside the box

4 The matrices **A** and **B** are such that

$$\mathbf{A} = \begin{bmatrix} 2 & a & 3 \\ 0 & -2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -3 \\ -2 & 4a \\ 0 & 5 \end{bmatrix}$$

4 (a) Find the product **AB** in terms of a.

[2 marks]

4 (b) Find the determinant of **AB** in terms of a.

[1 mark]

Do not write outside the box

4 (c) Show that **AB** is singular when $a = -1$

[2 marks]

Turn over for the next question

5 (a) Show that

$$r^2(r+1)^2 - (r-1)^2r^2 = pr^3$$

where p is an integer to be found.

[1 mark]

Do not write outside the box

5 (b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

[3 marks]

Do not write outside the box

6 Anna has been asked to describe the transformation given by the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

She writes her answer as follows:

The transformation is a rotation about the x-axis through an angle of θ, where

$$\sin\theta = \frac{1}{2} \quad \text{and} \quad -\sin\theta = -\frac{1}{2}$$

$$\theta = 30°$$

Identify and correct the error in Anna's work.

[2 marks]

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7 Prove by induction that, for all integers $n \geq 1$, the expression $7^n - 3^n$ is divisible by 4

[4 marks]

Do not write outside the box

8 (a) Prove that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

[5 marks]

8 (b) Prove that the graphs of

$$y = \sinh x \quad \text{and} \quad y = \cosh x$$

do **not** intersect.

[3 marks]

Do not write outside the box

9 The quadratic equation $2x^2 + px + 3 = 0$ has two roots, α and β, where $\alpha > \beta$.

9 (a) (i) Write down the value of $\alpha\beta$.

[1 mark]

9 (a) (ii) Express $\alpha + \beta$ in terms of p.

[1 mark]

9 (b) Hence find $(\alpha - \beta)^2$ in terms of p.

[2 marks]

Do not write outside the box

9 (c) **Hence** find, in terms of p, a quadratic equation with roots $\alpha - 1$ and $\beta + 1$

[4 marks]

Do not write outside the box

10 (a) Show that the equation

$$y = \frac{3x - 5}{2x + 4}$$

can be written in the form

$$(x + a)(y + b) = c$$

where a, b and c are constants to be found.

[3 marks]

10 (b) Write down the equations of the asymptotes of the graph of

$$y = \frac{3x - 5}{2x + 4}$$

[2 marks]

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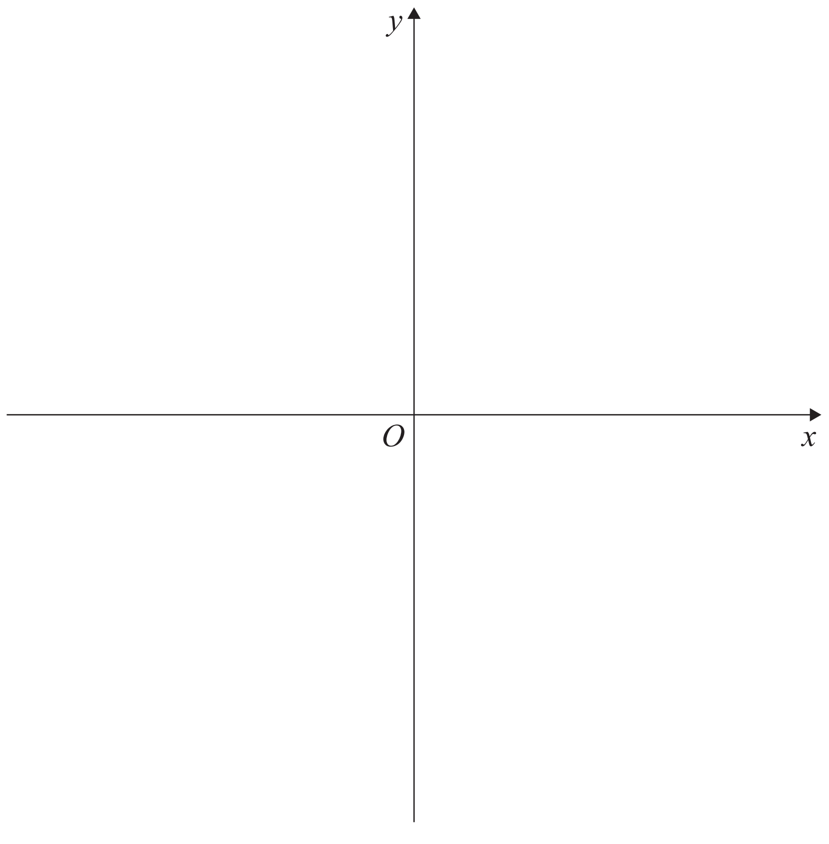
10 (c) Sketch, on the axes provided, the graph of

$$y = \frac{3x - 5}{2x + 4}$$

[3 marks]



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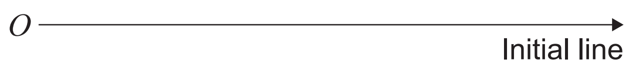
11 Sketch the polar graph of

$$r = \sinh\theta + \cosh\theta$$

for $0 \leq \theta \leq 2\pi$

[3 marks]



12 The mean value of the function f over the interval $1 \leq x \leq 5$ is m.

The graph of $y = \mathrm{g}(x)$ is a reflection in the x-axis of $y = \mathrm{f}(x)$.

The graph of $y = \mathrm{h}(x)$ is a translation of $y = \mathrm{g}(x)$ by $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Determine, in terms of m, the mean value of the function h over the interval $4 \leq x \leq 8$

[2 marks]

Turn over for the next question

13 Line l_1 has equation

$$\frac{x-2}{3} = \frac{1-2y}{4} = -z$$

and line l_2 has equation

$$\mathbf{r} = \begin{bmatrix} -7 \\ 4 \\ -2 \end{bmatrix} + \mu \begin{bmatrix} 12 \\ a+3 \\ 2b \end{bmatrix}$$

13 (a) In the case when l_1 and l_2 are parallel, show that $a = -11$ and find the value of b.

[4 marks]

13 (b) In a **different** case, the lines l_1 and l_2 intersect at exactly one point, and the value of b is 3

Find the value of a.

[5 marks]

14 (a) Given

$$\frac{x+7}{x+1} \leq x+1$$

show that

$$\frac{(x+a)(x+b)}{x+c} \geq 0$$

where a, b, and c are integers to be found.

[4 marks]

14 (b) Briefly explain why this statement is incorrect.

$$\frac{(x+p)(x+q)}{x+r} \geq 0 \Leftrightarrow (x+p)(x+q)(x+r) \geq 0$$

[1 mark]

14 (c) Solve

$$\frac{x+7}{x+1} \leq x+1$$

[2 marks]

Turn over for the next question

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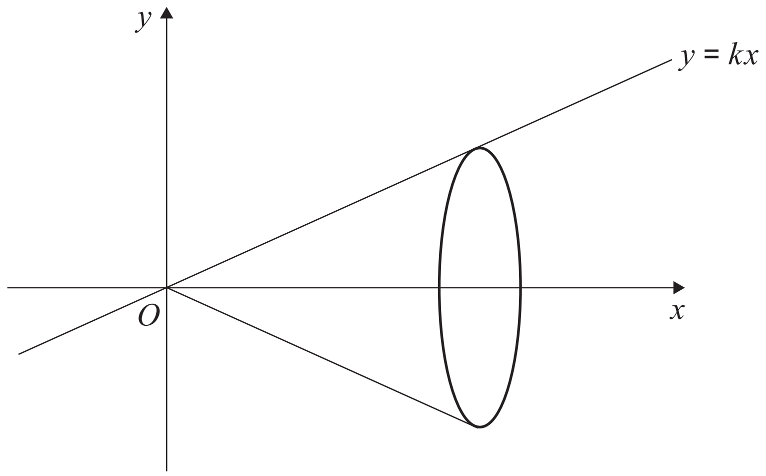
15 A segment of the line $y = kx$ is rotated about the x-axis to generate a cone with vertex O.

The distance of O from the centre of the base of the cone is h.

The radius of the base of the cone is r.



15 (a) Find k in terms of r and h.

[1 mark]

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15 (b) Use calculus to prove that the volume of the cone is

$$\frac{1}{3}\pi r^2 h$$

[3 marks]

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16 **A** and **B** are non-singular square matrices.

16 (a) Write down the product $\mathbf{AA}^{-1}$ as a single matrix.

[1 mark]

16 (b) **M** is a matrix such that $\mathbf{M} = \mathbf{AB}$.

Prove that $\mathbf{M}^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

[3 marks]

17 The polar equation of the circle C is

$$r = a(\cos\theta + \sin\theta)$$

Find, in terms of a, the radius of C.

Fully justify your answer.

[4 marks]

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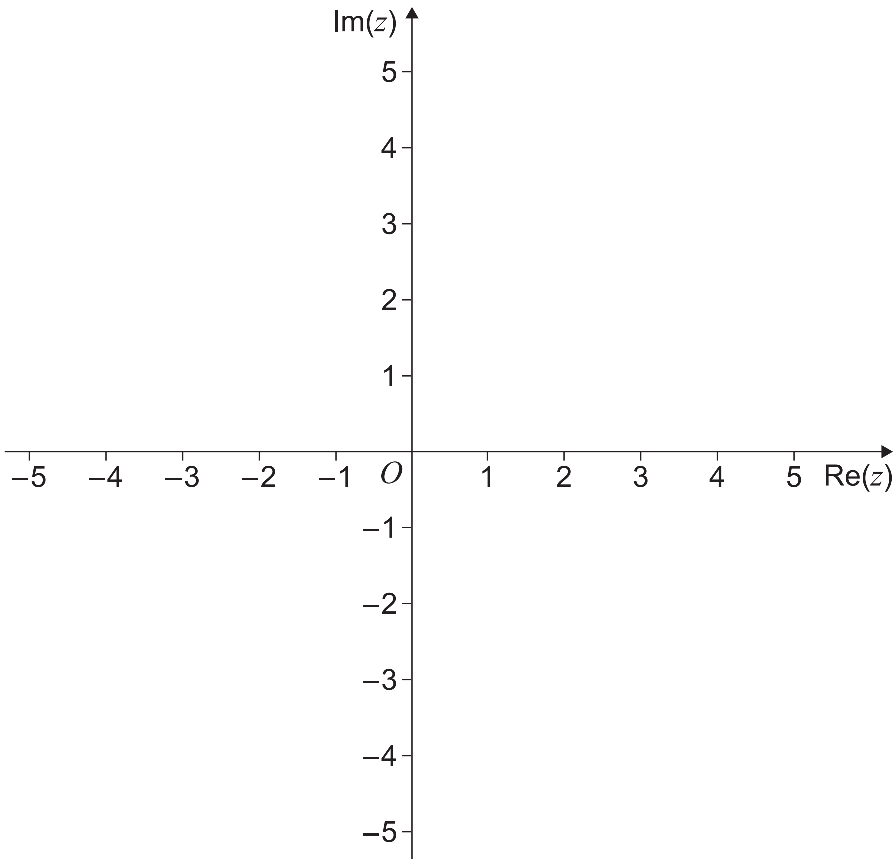
18 The locus of points L_1 satisfies the equation $|z| = 2$

The locus of points L_2 satisfies the equation $\arg(z + 4) = \frac{\pi}{4}$

18 (a) Sketch L_1 on the Argand diagram below.

[1 mark]



18 (b) Sketch L_2 on the Argand diagram above.

[1 mark]

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18 (c) The complex number $a + ib$, where a and b are real, lies on L_1

The complex number $c + id$, where c and d are real, lies on L_2

Calculate the least possible value of the expression

$$(c - a)^2 + (d - b)^2$$

[3 marks]

END OF QUESTIONS

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