L6 Further Mathematics

January Exam

Paper 1 (Teacher X)

January 2024

2023-2024

Duration: **1 hour 4 minutes**

Total number of marks: 51

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is included prior to each section of questions.

The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

1.

(a) The matrix **P** is given by $\mathbf{P} = \begin{pmatrix} 1 & 0 & -2 & 2 \\ 4 & 2 & -2 & 3 \end{pmatrix}$.

(i) Write down the dimensions of **P**. **[1]**

(ii) Write down the transpose of **P**. **[1]**

(b) The matrices **Q**, **R** and **S** are given by $\mathbf{Q} = \begin{pmatrix} 1 & 2 \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} 3 & -4 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{S} = \begin{pmatrix} 3 & -2 \end{pmatrix}$.

Write down the sum of the two of these matrices which are conformable for addition. **[1]**

(c) The dimensions of matrix **A** are 4 by 5. The matrices **A** and **B** are conformable for multiplication so that the matrix **C** = **BA** can be formed. The matrix **C** has 6 rows.

(i) Write down the number of columns that **C** has. **[1]**

(ii) Write down the dimensions of **B**. **[1]**

(iii) Explain whether the matrix **AB** can be formed. **[1]**

(d) Find the value of c for which $\begin{pmatrix} -2 & 3 \\ 6 & 10 \end{pmatrix}\begin{pmatrix} c & 5 \\ 10 & 13 \end{pmatrix} = \begin{pmatrix} c & 5 \\ 10 & 13 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 6 & 10 \end{pmatrix}$. **[2]**

2.

The lines L_1 and L_2 have the following equations.

$$L_1 : \mathbf{r} = \begin{pmatrix} -5 \\ 6 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ -2 \end{pmatrix}$$

$$L_2 : \mathbf{r} = \begin{pmatrix} 24 \\ 1 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$$

(a) Show that L_1 and L_2 intersect, giving the position vector of the point of intersection. **[5]**

(b) Find the equation of the line which intersects L_1 and L_2 and is perpendicular to both. Give your answer in cartesian form. **[3]**

3.

(i) $$\mathbf{P} = \begin{pmatrix} k & -2 & 7 \\ -3 & -5 & 2 \\ k & k & 4 \end{pmatrix}$$ where k is a constant

Show that **P** is non-singular for all real values of k.

(4)

(ii) $$\mathbf{Q} = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$$

The matrix **Q** represents a linear transformation T

Under T, the point $A(a, 2)$ and the point $B(4, -a)$, where a is a constant, are transformed to the points A' and B' respectively.

Given that the distance $A'B'$ is $\sqrt{58}$, determine the possible values of a.

(5)

4.

$$\mathbf{M} = \begin{pmatrix} -2 & 5 \\ 6 & k \end{pmatrix}$$

where k is a constant.

Given that

$$\mathbf{M}^2 + 11\mathbf{M} = a\mathbf{I}$$

where a is a constant and **I** is the 2×2 identity matrix,

(a) (i) determine the value of a

(ii) show that $k = -9$

(3)

(b) Determine the equations of the invariant lines of the transformation represented by **M**.

(6)

(c) State which, if any, of the lines identified in (b) consist of fixed points, giving a reason for your answer.

(1)

5.

The function f is defined by

$$f(n) = 3^{3n+1} + 2^{3n+4} \qquad (n \in \mathbb{Z}^+)$$

Prove by induction that $f(n)$ is divisible by 19 for $n \geq 1$

[6 marks]

6.

A stone of mass 0.5 kg is projected vertically upwards with a speed $U\,\text{m s}^{-1}$ from a point A. The point A is 2.5 m above horizontal ground.

The speed of the stone as it hits the ground is $25\,\text{m s}^{-1}$

The motion of the stone from the instant it is projected from A until the instant it hits the ground is modelled as that of a particle moving freely under gravity.

(a) Use the model and the principle of conservation of mechanical energy to find the value of U.

(4)

In reality, the stone will be subject to air resistance as it moves from A to the ground.

(b) State how this would affect your answer to part (a).

(1)

The ground is soft and the stone sinks a vertical distance d cm into the ground. The resistive force exerted on the stone by the ground is modelled as a constant force of magnitude 2000 N and the stone is modelled as a particle.

(c) Use the model and the work-energy principle to find the value of d, giving your answer to 3 significant figures.

(5)