L6 Further Mathematics

January Exam

Paper 1 (Teacher X)

January 2025

2024-2025

Duration: **1 hour 9 minutes**

Total number of marks: **55**

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is included prior to each section of questions.

The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

1.

Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 4 & -3 \\ -2 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & -5 \\ 0 & 1 \end{pmatrix}$.

(a) Find $2\mathbf{A} - 4\mathbf{B}$. **[2]**

(b) Write down the matrix **C** such that $\mathbf{AC} = 2\mathbf{A}$. **[1]**

(c) Find the value of det **A**. **[1]**

(d) **In this question you must show detailed reasoning.**

Use $\mathbf{A}^{-1}$ to solve the equations $4x - 3y = 7$ and $-2x + 2y = 9$. **[3]**

2.

(a) **(i)** Find $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$. **[1]**

(ii) State a geometrical relationship between the answer to part **(a)(i)** and the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$. **[1]**

(iii) Verify the relationship stated in part **(a)(ii)**. **[2]**

(b) Find the angle between the vectors $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $4\mathbf{i} - \mathbf{j} + 8\mathbf{k}$. **[3]**

3.

The line through points $A(8, -7, -2)$ and $B(11, -9, 0)$ is denoted by L_1.

(a) Find a vector equation for L_1. **[2]**

(b) Determine whether the point $(26, -19, -14)$ lies on L_1. **[2]**

The line L_2 passes through the origin, O, and intersects L_1 at the point C. The lines L_1 and L_2 are perpendicular.

(c) By using the fact that C lies on L_1, find a vector equation for L_2. **[4]**

4.

Three transformations, T_A, T_B and T_C, are represented by the matrices **A**, **B** and **C** respectively.

You are given that $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(a) Find the matrix which represents the inverse transformation of T_A. **[1]**

(b) By considering matrix multiplication, determine whether T_A followed by T_B is the same transformation as T_B followed by T_A. **[2]**

Transformations R and S are each defined as being the result of successive transformations, as specified in the table.

Transformation	First transformation	followed by
R	T_A followed by T_B	T_C
S	T_A	T_B followed by T_C

(c) Explain, using a property of matrix multiplication, why R and S are the same transformations. **[2]**

A quadrilateral, Q, has vertices D, E, F and G in anticlockwise order from D. Under transformation R, Q's image, Q', has vertices D', E', F' and G' (where D' is the image of D, etc). The area of Q, in suitable units, is 5.

You are given that $\det \mathbf{C} = a^2 + 1$ where a is a real constant.

(d) (i) Determine the order of the vertices of Q', starting anticlockwise from D'. **[2]**

(ii) Find, in terms of a, the area of Q'. **[1]**

5.

Prove by induction that $11 \times 7^n - 13^n - 1$ is divisible by 3, for all integers $n \geqslant 0$. **[5]**

6.

The matrix **M** represents the transformation T, and is given by

$$\mathbf{M} = \begin{bmatrix} 3 & -1 \\ -2 & 6 \end{bmatrix}$$

(a)

Show that the only invariant point under T is the origin.

[3 marks]

(b)

The line L_1 has equation $y = x + 1$

The transformation T maps the line L_1 onto the line L_2

Find the equation of L_2 in the form $y = mx + c$

[5 marks]

Mechanics

7.

A car of mass 1000 kg moves in a straight line along a horizontal road at a constant speed of $72\,\text{km h}^{-1}$

- The resistance to the motion of the car is modelled as a constant force of magnitude 900 N

The engine of the car is working at a constant rate of P kW.

Using the model,

(a) find the value of P.

(3)

The car now travels in a straight line up a road which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{2}{49}$

- In a refined model, the resistance to the motion of the car from non-gravitational forces is now modelled as a force of magnitude $20v$ newtons, where $v\,\text{m s}^{-1}$ is the speed of the car

At the instant when the engine of the car is working at a constant rate of 30 kW and the car is moving up the road at $10\,\text{m s}^{-1}$, the acceleration of the car is $a\,\text{m s}^{-2}$

Using the refined model,

(b) find the value of a.

(4)

Later on, when the engine of the car is again working at a constant rate of 30 kW, the car is moving up the road at a constant speed $U\,\text{m s}^{-1}$

Using the refined model,

(c) find the value of U.

(5)