

L6 Further Mathematics

January Exam

Paper 2 (Teacher Y)

January 2024

2023-2024

Duration: 59 minutes

Total number of marks: 47

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is included prior to each section of questions.

1.

In this question you must show detailed reasoning.

In this question the principal argument of a complex number lies in the interval $[0, 2\pi)$.

Complex numbers z_1 and z_2 are defined by $z_1 = 3 + 4i$ and $z_2 = -5 + 12i$.

(a) Determine $z_1 z_2$, giving your answer in the form $a + bi$. [2]

(b) Express z_2 in modulus-argument form. [3]

(c) Verify, by direct calculation, that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$. [3]

2.

Given that a cubic equation has three distinct roots that all lie on the same straight line in the complex plane,

(a) describe the possible lines the roots can lie on. (2)

$$f(z) = 8z^3 + bz^2 + cz + d$$

where b , c and d are real constants.

The roots of $f(z)$ are distinct and lie on a straight line in the complex plane.

Given that one of the roots is $\frac{3}{2} + \frac{3}{2}i$

(b) state the other two roots of $f(z)$ (1)

$$g(z) = z^3 + Pz^2 + Qz + 12$$

where P and Q are real constants, has 3 distinct roots.

The roots of $g(z)$ lie on a different straight line in the complex plane than the roots of $f(z)$

Given that

- $f(z)$ and $g(z)$ have one root in common
- one of the roots of $g(z)$ is -4

(c) (i) write down the value of the common root, (1)

(ii) determine the value of the other root of $g(z)$ (3)

(d) Hence solve the equation $f(z) = g(z)$ (4)

3.

(i) Shade, on an Argand diagram, the set of points for which

$$|z - 3| \leq |z + 6i| \quad (3)$$

(ii) Determine the exact complex number w which satisfies both

$$\arg(w - 2) = \frac{\pi}{3} \quad \text{and} \quad \arg(w + 1) = \frac{\pi}{6} \quad (6)$$

4.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) The quartic equation

$$z^4 + 5z^2 - 30 = 0$$

has roots p, q, r and s .

Without solving the equation, determine the quartic equation whose roots are

$$(3p - 1), (3q - 1), (3r - 1) \text{ and } (3s - 1)$$

Give your answer in the form $w^4 + aw^3 + bw^2 + cw + d = 0$, where a, b, c and d are integers to be found.

(5)

(ii) The roots of the cubic equation

$$4x^3 + nx + 81 = 0 \quad \text{where } n \text{ is a real constant}$$

are $\alpha, 2\alpha$ and $\alpha - \beta$

Determine

(a) the values of the roots of the equation, (5)

(b) the value of n . (2)

5.

A music lover has 30 CDs arranged in a random order in a line on a shelf. Of these CDs, 7 are classed as Baroque, 10 as Classical and 13 as Romantic.

(a) Determine the probability that all 7 Baroque CDs are next to each other. [3]

(b) Determine the probability that, of the 10 CDs furthest to the left on the shelf, at least 6 are Baroque. [4]