L6 Further Maths January Exam Teacher Y Mock 23-24 SOLUTIONS [47]

1.

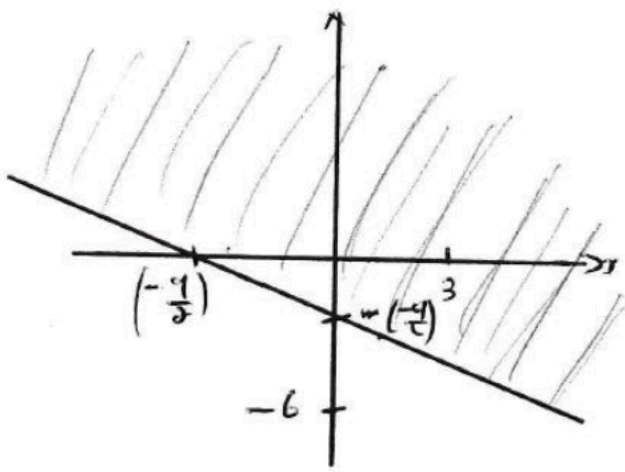
(a)		DR $z_1z_2 = (3+4i)(-5+12i) = -15+36i-20i-48$	M1	1.1	Attempt at expansion (4 terms soi) using $i^2 = -1$	DR – Need to see at least one line of expanded terms before answer
		$= -63+16i$	A1 [2]	1.1		
(b)		DR $\lvert z_2\rvert = (\sqrt{(-5)^2+12^2} = \sqrt{169}) = 13$	B1	1.1	Not ± unless later corrected. Allow modulus of 13 for the B1 as long as no incorrect working	Treat attempt to write z_1 or z_1z_2 in mod/arg form as MR so **B0M1A1** available
		$\tan^{-1}\left(\frac{12}{-5}\right)$	M1	1.1	Evidence of using trigonometry towards finding the correct angle, perhaps by finding a related angle. Treat $\tan^{-1}\left(\frac{12}{5}\right)$ as such evidence for M1 but not $\tan^{-1}\left(-\frac{5}{12}\right)$ or $\tan^{-1}\left(\frac{5}{12}\right)$ unless supported eg by a diagram or by working leading to correct answer.	
		$\therefore z_2 = 13(\cos 1.97 + i\sin 1.97)$ (3 sf)	A1 [3]	2.5	For argument accept awrt 1.97 only. Do not accept answers not written correctly in mod-arg form. Do not accept –1.18 or –4.32 as argument. Answer must be in radians for A1. Accept $[r, \theta]$ or r cis θ	Is 1.965587446… eg do not accept the following $13\cos 1.97 + 13i\sin 1.97$ $13(\cos 4.32 - i\sin 4.32)$ NB $z_1 = 5(\cos 0.927 + i\sin 0.927)$ $z_1z_2 = 65(\cos 2.89 + i\sin 2.89)$
(c)		DR $(\arg(z_1z_2) =)\tan^{-1}\left(\frac{16}{-63}\right)$	M1	1.1	**Using trigonometry** to find the argument. Do not accept any other form unless supported by clear evidence (eg diagram)	This mark may be awarded if z_1z_2 was incorrect from **(a)**.
		$\arg(z_1)+\arg(z_2) = \tan^{-1}\left(\frac{4}{3}\right) + 1.965587\ldots$ $= 0.927295\ldots + 1.965587\ldots$ $= 2.89288\ldots$	M1	2.1	Attempt to calculate RHS using their values (either value could have been found earlier but both must be in $[0, 2\pi)$).	Could accept 0.927… as evidence of arctan(4/3)
		$\arg(z_1z_2) = -0.2487099\ldots + \pi = 2.892882\ldots$ so they are equal	A1 [3]	2.2a	Accept rounding to 3 sf or better but rounding must be correct (e.g. 0.927 + 1.96 = 2.89 would score **A0**).	Answer must be in radians for A1. If MR z_1 or z_1z_2 in part (b) then full credit available for a correct solution here.

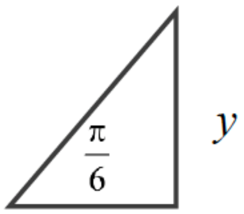
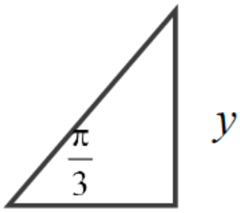
2.

8(a)	The real axis. Horizontal line through (0, 0) Line $y = 0$ Accept on a diagram	The other possibility is that all three roots have the same real part so lie on a vertical line/perpendicular to the real axis/parallel to the imaginary axis Line $x = k$ where k is a real number Accept on a diagram	**B1** **B1**
			(2)
(b)	Other roots are $\frac{3}{2}$ and $\frac{3}{2}-\frac{3}{2}\text{i}$		**B1**
			(1)
(c)(i)	Common root must be $\frac{3}{2}$		**B1**
			(1)
(ii)	Sets product of roots = – 12 using their $\frac{3}{2}\times -4\times\alpha = -12$ Or $\text{g}(z)=\left(z-\frac{3}{2}\right)(z+4)(z-\alpha)$		**M1**
	Solves to find a value of the third root their $\frac{3}{2}\times -4\times\alpha = -12 \Rightarrow \alpha = 2$ Or $\text{g}(z)=\left(z-\frac{3}{2}\right)(z\pm 4)(z-\alpha)\Rightarrow -\frac{3}{2}\times 4\times -\alpha = 12 \Rightarrow \alpha = 2$		**M1** **A1**
			(3)
(d)	$8\left\{z-\frac{3}{2}\right\}\left(z-\frac{3}{2}-\frac{3}{2}\text{i}\right)\left(z-\frac{3}{2}+\frac{3}{2}\text{i}\right)=8\left\{z-\frac{3}{2}\right\}\left(z^2-3z+\frac{9}{2}\right)$ Or $b=-8\left(\frac{3}{2}+\frac{3}{2}+\frac{3}{2}\text{i}+\frac{3}{2}-\frac{3}{2}\text{i}\right)=\ldots\{-36\}$ $c=8\left[\left(\frac{3}{2}\times\left(\frac{3}{2}+\frac{3}{2}\text{i}\right)\right)+\left(\frac{3}{2}\times\left(\frac{3}{2}-\frac{3}{2}\text{i}\right)\right)+\left(\left(\frac{3}{2}+\frac{3}{2}\text{i}\right)\times\left(\frac{3}{2}-\frac{3}{2}\text{i}\right)\right)\right]=\ldots\{72\}$ $d=-8\left[\frac{3}{2}\times\left(\frac{3}{2}+\frac{3}{2}\text{i}\right)\times\left(\frac{3}{2}-\frac{3}{2}\text{i}\right)\right]=\ldots\{-54\}$		**M1**

	$\mathrm{f}(z)=\mathrm{g}(z)\Rightarrow 8\left(z-\frac{3}{2}\right)\left(z^2-3z+\frac{9}{2}\right)=\left(z-\frac{3}{2}\right)(z+4)(z-2)$ $\Rightarrow 8z^2-24z+36=(z+4)(z-2)$ Or Either $\mathrm{g}(z)=\left(z-\frac{3}{2}\right)(z+4)(z-2)=\ldots$ or $P=-\left(\frac{3}{2}-4+2\right)=\ldots\left\{\frac{1}{2}\right\}$ and $Q=\left(\frac{3}{2}\times-4\right)+\left(\frac{3}{2}\times 2\right)+(2\times-4)=\ldots\{-11\}$ to find g(z) and sets their f(z) = their g(z) $8z^3-36z^2+72z-54=z^3+\frac{1}{2}z^2-11z+12$	**M1**
	$7z^2-26z+44=0\Rightarrow z=\ldots$ or $7z^3-\frac{73}{2}z^2+83z-66=0\Rightarrow z=\ldots$	**M1**
	So solutions are $\frac{3}{2}$, $\frac{13\pm \mathrm{i}\sqrt{139}}{7}$	**A1**
		(4)

3.

(i)		M1
		A1
		B1

(ii)	$m=\tan\left(\frac{\pi}{3}\right)\left\{=\sqrt{3}\right\}$ and $y-0=m(x-2)$ leads to $y-0=\sqrt{3}(x-2)$ or $y=\sqrt{3}x-2\sqrt{3}$ $m=\tan\left(\frac{\pi}{6}\right)\left\{=\frac{\sqrt{3}}{3}\right\}$ and $y-0=m(x-(-1))$ leads to $y-0=\frac{\sqrt{3}}{3}(x-(-1))$ or $y=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}$	M1 A1 A1
	$\sqrt{3}x-2\sqrt{3}=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}\Rightarrow x=\ldots$	M1
	$y=\sqrt{3}\left(\frac{7}{2}\right)-2\sqrt{3}=\ldots$	M1
	$\{w=\}\frac{7}{2}+\frac{3\sqrt{3}}{2}\mathrm{i}$	A1
	Alternative $\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}=\frac{y}{x_{-1}}$ and $\tan\left(\frac{\pi}{3}\right)=\sqrt{3}=\frac{y}{x_2}$	M1
	 $x_{-1}=\sqrt{3}y$  $x_2=\frac{\sqrt{3}}{3}y$	A1 A1
	$y\sqrt{3}=y\frac{\sqrt{3}}{3}+3\Rightarrow y=\ldots$	M1
	Uses $x=y\sqrt{3}-1$ or $x=\frac{\sqrt{3}}{3}y+2$ with their value of y leading to a value for x	M1
	$(w=)\frac{7}{2}+\frac{3\sqrt{3}}{2}\mathrm{i}$	A1
		(6)

4.

0(i)	$w = 3z - 1 \Rightarrow z = \frac{w+1}{3}$	B1
	$\left(\frac{w+1}{3}\right)^4 + 5\left(\frac{w+1}{3}\right)^2 - 30 = 0$	M1
	$\frac{1}{81}\left(w^4 + 4w^3 + 6w^2 + 4w + 1\right) + \frac{5}{9}\left(w^2 + 2w + 1\right) - 30 = 0$ leading to $w^4 + aw^3 + bw^2 + cw + d$ $\{= 0\}$	M1
	$w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$	A1 A1
		(5)
	Alternative $p + q + r + s = 0, \quad pq + pr + ps + qr + qs + rs = 5$ $pqr + pqs + prs + qrs = 0, \quad pqrs = -30$	B1
	New sum $= 3(p+q+r+s) - 4 = \ldots\{-4\}$ New pair sum $= 9(pq + pr + ps + qr + qs + rs) - 9(p+q+r+s) + 6 = \ldots\{51\}$ New triple sum $= 27(pqr + pqs + prs + qrs) - 18(pq + pr + ps + qr + qs + rs)$ $+6(p+q+r+s) - 4 = \ldots\{-94\}$ $= 81(pqrs) - 27(pqr + pqs + prs + qrs)$ New product $+9(pq + pr + ps + qr + qs + rs) - 3(p+q+r+s) + 1$ $= \ldots\{-2384\}$	M1
	Applies $w^4 - (\text{new sum})w^3 + (\text{new pair sum})w^2 - (\text{new triple sum})w$ $+ (\text{new product}) = 0$	M1
	$w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$	A1 A1

(ii) (a)	$\alpha + 2\alpha + \alpha - \beta = 0$ and $\alpha \times 2\alpha \times (\alpha - \beta) = -\frac{81}{4}$	M1 A1
	Solves simultaneously e.g. $4\alpha - \beta = 0 \Rightarrow \beta = 4\alpha$ $2\alpha^2(\alpha - 4\alpha) = -\frac{81}{4} \Rightarrow \alpha^3 = \frac{27}{8} \Rightarrow \alpha = \ldots$	M1
	Uses their values $\alpha = \frac{3}{2}\ \beta = 6$ to find the roots $\alpha, 2\alpha, \alpha - \beta$	M1
	Roots 1.5, 3, – 4.5	A1
(ii) (b)	$n = [(1.5 \times 3) + (1.5 \times -4.5) + (3 \times -4.5)] \times 4$ Or Multiplies out $(x-3)\left(x - \frac{3}{2}\right)\left(x + \frac{9}{2}\right)$ or $(x-3)(2x-3)(2x+9)$ to achieve the form $4x^3 + \ldots$	M1
	$n = -63$ cso (must have correct roots in (a)	A1

5.

(a)	Class Baroque CDs as single unit $24! \times 7!$ $(= 6.2\times10^{23} \times 5040)$ $\div 30! = 1.18\times10^{-5} = 0.000\ 011\ 8$	**M1** **A1** **A1** **[3]**	3.1b 1.1 1.1	e.g. 23!×7! seen, with or without other terms, *or* 24! (with 7! omitted) These, and no other terms, in numerator (allow even if no denominator) Awrt 1.2×10^{-5}, or $\frac{1}{84825}$
(b)	6: ${}^7C_6 \times {}^{23}C_4$ $(= 7\times8855$ or $61\ 985)$ 7: $1 \times {}^{23}C_3$ $(= 1771)$ Add, and divide by ${}^{30}C_{10}$ $(= 30\ 045\ 015)$ $= \frac{4}{1885}$ or 0.002 12 (0.002 122 ...)	**M1*** **A1** **depM1** **A1** **[4]**	2.1 1.1 3.1b 1.1	Clear attempt at one (allow for ${}^7C_6 \times {}^{23}C_4 \times$ other things), allow ${}^{10}C_6 \times \ldots$ Both expressions correct $[= \frac{7}{3393} + \frac{1}{16965}]$ Needs two terms, allow dividing by ${}^{30}P_{10}$ if consistent Any equivalent exact fraction, or 0.00212 or better SC: $B(10, \frac{7}{30})$, 0.014(0): B1 max SC: $({}^7P_6 \times {}^{23}P_4) + ({}^7P_7 \times {}^{23}P_3)$, M1; × 20!/30!, M1 (same as $\div {}^{30}P_{10}$)
OR	${}^7C_6 \times [{}^{10}C_4 + {}^{10}C_3\times13 + {}^{10}C_2\times{}^{13}C_2 + 10\times{}^{13}C_3 + {}^{13}C_4] + [{}^{10}C_3 + {}^{10}C_2\times13 + 10\times{}^{13}C_2 + {}^{13}C_3] = 7\times(210+1560+3510+2860+715) + (120+585+780+286)$			