

L6 Further Mathematics Mock

Paper 1 (Teacher X)

January 2020

2019-2020

Duration: 1 hour 15 minutes

Total number of marks: 60

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is included prior to each section of questions.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

Pure [40]

1.

The matrix $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$ and h and k are numbers such that $\mathbf{A}^2 = h\mathbf{A} + k\mathbf{I}$, where \mathbf{I} is the identity matrix. Find h and k .

[5 marks]

2.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}$$

(a) Show that the matrix \mathbf{M} is non-singular.

(2)

The transformation T of the plane is represented by the matrix \mathbf{M} .

The triangle R is transformed to the triangle S by the transformation T .

Given that the area of S is 63 square units,

(b) find the area of R .

(2)

[4 marks]

3.

\mathbf{S} is a singular matrix such that

$$\det \mathbf{S} = \begin{vmatrix} a & a & x \\ x-b & a-b & x+1 \\ x^2 & a^2 & ax \end{vmatrix}$$

Express the possible values of x in terms of a and b .

[7 marks]

4.

Let two lines L_1 and L_2 be given by:

$$L_1: r = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \\ k \end{pmatrix} \quad \text{and} \quad L_2: \frac{x-3}{4} = 2 - y = \frac{3z+5}{6}$$

(a) Find the value of k such that the lines intersect. [5]

(b) Given that $k = 1$, find the equation of the line in vector form that is perpendicular to L_1 and L_2 and passes through the origin. [2]

(c) Find the values of k such that the angle between the lines is 45° . Give your answer to 3 significant figures [5]

[12 marks]

5.

The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

(a) Prove by induction that, for all integers $n \geq 1$,

$$\mathbf{A}^n = \begin{bmatrix} 1 & 3^n - 1 \\ 0 & 3^n \end{bmatrix}$$

[4 marks]

(b) Find all invariant lines under the transformation matrix \mathbf{A} .

Fully justify your answer.

[6 marks]

(c) Find a line of invariant points under the transformation matrix \mathbf{A} .

[2 marks]

[12 marks]

Mechanics [20]

7.

A boy drags a sledge in a straight line along horizontal ground by means of a rope attached to the sledge. The rope makes an angle of 25° with the horizontal and the tension in the rope, T N, is constant. The work done by the tension in moving the sledge 75 m is 4000 J. Calculate the value of T .

[3]

[3 marks]

8.

A car, of mass 1200 kg, moves on a straight horizontal road where it has a maximum speed of 40 m s^{-1}

When the car travels at a speed of $v \text{ m s}^{-1}$ it experiences a resistance force which can be modelled as being of magnitude $30v$ newtons.

(a) Show that the power output of the car is 48 000 W, when it is travelling at its maximum speed.

[3 marks]

(b) Find the maximum acceleration of the car when it is travelling at a speed of 25 m s^{-1}

[4 marks]

[7 marks]

9.

The masses of two particles A and B are 4 kg and 3 kg respectively. The particles are moving towards each other along a straight line on a smooth horizontal surface. A has speed 8 m s^{-1} and B has speed 10 m s^{-1} before they collide. The kinetic energy lost due to the collision is 121.5 J.

(i) Find the speed and direction of motion of each particle after the collision.

[8]

(ii) Find the coefficient of restitution between A and B .

[2]

[10 marks]