

L6 Further Mathematics Mock

Paper 1 (Teacher X)

January 2022

2021-2022

Duration: 1 hour 15 minutes

Total number of marks: 56

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is included prior to each section of questions.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

1.

By using the determinant of an appropriate matrix, or otherwise, find the value of k for which the simultaneous equations

$$2x - y + z = 7,$$

$$3y + z = 4,$$

$$x + ky + kz = 5,$$

do not have a unique solution for x , y and z .

[5]

2.

Two matrices \mathbf{A} and \mathbf{B} satisfy the equation

$$\mathbf{AB} = \mathbf{I} + 2\mathbf{A}$$

where \mathbf{I} is the identity matrix and $\mathbf{B} = \begin{bmatrix} 3 & -2 \\ -4 & 8 \end{bmatrix}$

Find \mathbf{A} .

[3 marks]

3.

Prove by induction that $f(n) = n^3 + 3n^2 + 8n$ is divisible by 6 for all integers $n \geq 1$

[7 marks]

4.

Line l_1 has Cartesian equation

$$x - 3 = \frac{2y + 2}{3} = 2 - z$$

(a) Write the equation of line l_1 in the form

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$$

where λ is a parameter and \mathbf{a} and \mathbf{b} are vectors to be found.

[2 marks]

(b) Line l_2 passes through the points $P(3, 2, 0)$ and $Q(n, 5, n)$, where n is a constant.

(b) (i) Show that the lines l_1 and l_2 are **not** perpendicular.

[3 marks]

(b) (ii) Explain briefly why lines l_1 and l_2 cannot be parallel.

[2 marks]

(b) (iii) Given that θ is the acute angle between lines l_1 and l_2 , show that

$$\cos \theta = \frac{p}{\sqrt{34n^2 + qn + 306}}$$

where p and q are constants to be found.

[3 marks]

5.

Consider the matrix $M = \begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix}$.

(a) Find all the invariant points under the matrix M .

(b) Find all the invariant lines under the matrix M .

[8 marks]

6.

(i) Write down the matrix, A , that represents an enlargement, centre $(0, 0)$, with scale factor $\sqrt{2}$.

[1]

(ii) The matrix B is given by $B = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by B .

[3]

(iii) Given that $C = AB$, show that $C = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$.

[1]

(iv) Draw a diagram showing the unit square and its image under the transformation represented by C .

[2]

(v) Write down the determinant of C and explain briefly how this value relates to the transformation represented by C .

[2]

7.

A particle, P , of mass m kg is projected with speed 5 m s^{-1} down a line of greatest slope of a rough plane. The plane is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{3}{5}$

The total resistance to the motion of P is a force of magnitude $\frac{1}{5} mg$

Use the work-energy principle to find the speed of P at the instant when it has moved a distance 8 m down the plane from the point of projection.

(7)

8.

A car of mass 1000 kg has a maximum speed of 40 m s^{-1} when travelling on a straight horizontal race track.

The maximum power output of the car's engine is 48 kW

The total resistance force experienced by the car can be modelled as being proportional to the car's speed.

Find the maximum possible acceleration of the car when it is travelling at 25 m s^{-1} on the straight horizontal race track.

Fully justify your answer.

[7 marks]