L6 Further Mathematics Mock Teacher X 21-22 SOLUTIONS [56]

1.

Either	M1		Consider determinant of coefficients of LHS
	M1		Sensible attempt at evaluating any 3×3 det
$4k-4$	A1		Obtain correct answer a.e.f. unsimplified
	M1		Equate det to 0
$k=1$	A1ft	5	Obtain $k=1$, ft provided all M's awarded
Or	M1		Eliminate either x or y
	A1		Obtain correct equation
	M1		Eliminate 2^{nd} variable
	A1		Obtain correct linear equation
	A1		Deduce that $k=1$
		5	

2.

Uses factorisation or pre-multiplication to isolate $\boldsymbol{A}$	3.1a	M1	$\boldsymbol{AB}-2\boldsymbol{A}=\boldsymbol{I}$ $\boldsymbol{A}(\boldsymbol{B}-2\boldsymbol{I})=\boldsymbol{I}$
Deduces $\boldsymbol{A}$ in terms of $\boldsymbol{B}$ and $\boldsymbol{I}$. Could be implied by sight of $\begin{bmatrix}1 & -2\\ -4 & 6\end{bmatrix}$ with attempt to invert.	2.2a	A1	$\boldsymbol{A}=(\boldsymbol{B}-2\boldsymbol{I})^{-1}$
Obtains correct matrix $\boldsymbol{A}$.	1.1b	A1	$\boldsymbol{A}=\begin{bmatrix}1 & -2\\ -4 & 6\end{bmatrix}^{-1}$ $\boldsymbol{A}=\frac{1}{-2}\begin{bmatrix}6 & 2\\ 4 & 1\end{bmatrix}$ $\boldsymbol{A}=\begin{bmatrix}-3 & -1\\ -2 & -0.5\end{bmatrix}$

Sets up four equations with at least three correct.	3.1a	M1	Let $\boldsymbol{A}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ $\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}3 & -2\\ -4 & 8\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+2\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ $3a-4b=1+2a$ and $3c-4d=0+2c$ and $-2a+8b=0+2b$ and $-2c+8d=1+2d$
Deduces at least two correct elements of $\boldsymbol{A}$. Note: Two correct elements from just two correct equations can score M1A1.	2.2a	A1	$a=4b+1$ and $c=4d$ $6b=2a$ and $6d=2c+1$ $\therefore\ 3b=4b+1$ and $6d=2(4d)+1$ $-1=b$ and $-1=2d \Rightarrow d=-\frac{1}{2}$ $\therefore\ a=4\times-1+1$ and $c=4\times-\frac{1}{2}$ $a=-3$ and $c=-2$
Obtains correct matrix $\boldsymbol{A}$	1.1b	A1	$\therefore\ \boldsymbol{A}=\begin{bmatrix}-3 & -1\\ -2 & -0.5\end{bmatrix}$
Total		**3**	

3.

Demonstrates the result for $n = 1$ and states that it is true for $n = 1$	AO1.1b	B1	Let $n = 1$ then $f(1) = 12 = 2 \times 6$ so the result is true for $n = 1$ Assume the result is true for $n = k$:
Assumes the result true for $n = k$	AO2.4	M1	Then $f(k) = 6m$ for some integer m $f(k+1) = (k+1)^3 + 3(k+1)^2 + 8(k+1)$ $= k^3 + 3k^2 + 3k + 1 + 3(k^2 + 2k + 1) + 8(k+1)$
Obtains the difference between $f(k+1)$ and $f(k)$	AO3.1a	M1	$= k^3 + 6k^2 + 17k + 12$
Calculates the difference between $f(k+1)$ and $f(k)$correctly	AO1.1b	A1	$f(k+1) = k^3 + 6k^2 + 17k + 12 - (k^3 + 3k^2 + 8k)$ $= 6m + 3k^2 + 9k + 12$ $3k^2 + 9k + 12$ is a multiple of 3
Deduces that the difference is a multiple of 3	AO2.2a	M1	$3k^2 + 9k + 12 = 3(k^2 + 3k) + 12$ $k^2 + 3k = k(k+3)$ and one of k or $k+3$ is
Deduces that the difference is a multiple of 2	AO2.2a	M1	even $\therefore k^2 + 3k$ is even and $3(k^2 + 3k + 4)$ is an even multiple of 3 and hence divisible by 6
Completes a rigorous argument and explains how their argument proves the required result	AO2.1	R1	$\therefore f(k+1)$ is divisible by 6 if $f(k)$ is divisible by 6 We also know that $f(1)$ is divisible by 6, so by induction this completes the proof.
Total		**7**	

4.

13(a)	Rewrites l_1 in the general Cartesian form, or as a vector in terms of just one parameter. Or finds the position vector of a point on the line. Or finds a direction vector of l_1 Or writes three equations expressing x, y and z in terms of the parameter.	3.1a	M1	$\frac{x-3}{1} = \frac{y+1}{1.5} = \frac{z-2}{-1}$
	Writes l_1 in a correct vector form. Accept $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ in place of $\mathbf{r}$ NMS can score 2/2	1.1b	A1	$\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1.5 \\ -1 \end{pmatrix}$
13(b)(i)	Calculates the scalar product of their direction vectors of lines l_1 and l_2 Must not use a position vector as a direction vector.	3.1a	M1	
	Calculates a correct scalar product of a vector parallel to $\begin{pmatrix} 1 \\ 1.5 \\ -1 \end{pmatrix}$ with a vector parallel to $\begin{pmatrix} n-3 \\ 3 \\ n \end{pmatrix}$	1.1b	A1	$\begin{pmatrix} 1 \\ 1.5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} n-3 \\ 3 \\ n \end{pmatrix} = n - 3 + 4.5 - n = 1.5$
	Explains that, as the scalar product is not zero, then the lines are not perpendicular. Follow through their direction vectors if their scalar product is non-zero. Must follow M1. NMS scores 0/3	3.2a	E1F	The scalar product is not zero $\therefore$ lines l_1 and l_2 are not perpendicular

13(b)(ii)	Explains that two vectors are parallel if one is a multiple of the other. l_1 and l_2 need not be referred to explicitly. Possibly implied by $\boldsymbol{c} = k\boldsymbol{d}$ seen where $\boldsymbol{c}$ and $\boldsymbol{d}$ are their direction vectors.	2.4	E1F	If $\begin{pmatrix}2\\3\\-2\end{pmatrix}$ and $\begin{pmatrix}n-3\\3\\n\end{pmatrix}$ are parallel then $n-3=2$ and $n=-2$
	Demonstrates that, as n varies, the two direction vectors can never be a multiple of each other.	3.1a	B1	but n cannot be both 5 and -2 $\therefore$ l_1 and l_2 cannot be parallel
13(b)(iii)	Uses the scalar product to form an equation in n and $\cos\theta$ Follow through their direction vectors for lines l_1 and l_2	1.1a	M1	$\begin{pmatrix}2\\3\\-2\end{pmatrix}.\begin{pmatrix}n-3\\3\\n\end{pmatrix} = \sqrt{2^2+3^2+2^2} \times \sqrt{(n-3)^2+3^2+n^2} \times \cos\theta$
	Forms a correct equation in n and $\cos\theta$ Or gives a correct expression for $\cos\theta$ Accept a correct equation, or expression, for the supplementary angle.	1.1b	A1	$2(n-3)+9-2n = \sqrt{17} \times \sqrt{2n^2-6n+18} \times \cos\theta$
	Writes $\cos\theta = \frac{3}{\sqrt{34n^2-102n+306}}$	2.1	R1	$\cos\theta = \frac{3}{\sqrt{34n^2-102n+306}}$
	Total		**10**	

5.

(a) For an invariant point: $\begin{pmatrix}2 & -1\\4 & -3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$ **[M1]**

$2x - y = x$ and $4x - 3y = y$ **[A1]**

$y = 3x$ and $y = x$

The only point that satisfies these equations is $(0, 0)$ **[A1]**

(b) Let the invariant lines be $y = mx + c$ so any point has coordinates $(k, mk + c)$

$\begin{pmatrix}2 & -1\\4 & -3\end{pmatrix}\begin{pmatrix}k\\mk+c\end{pmatrix} = \begin{pmatrix}2k-mk+c\\4k-3mk-3c\end{pmatrix}$ **[M1]**

The image point must also lie on the line $y = mx + c$:

$4k - 3mk - 3c = m(2k - mk + c) + c$ must be true for all values of k and c.

So $k(4 - 3m - 2m - m^2) + c(-3 + m - 1) = 0$ **[A1] oe**

$m^2 - 5m + 4 = 0$ means $m = 4$ or $m = 1$ **[M1] solve their equation in *m***

When $m = 4$, $-3 + m - 1 = -3 + 4 - 1 = 0$ so c can take any value

When $m = 1$, $-3 + m - 1 = -3 + 1 - 1 \neq 0$ so when $m = 1$, $c = 0$

The invariant lines are $y = x$ and $y = 4x + c$ **[A1, A1]**

[8 marks]

6.

(i) $\begin{pmatrix}\sqrt{2} & 0\\0 & \sqrt{2}\end{pmatrix}$	B1	1	Correct matrix
(ii) Rotation (centre O), 45^0, clockwise	B1B1B1	3	Sensible alternatives OK, must be a single transformation
(iii)	B1	1	Matrix multiplication or combination of transformations
(iv) $\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}1\\1\end{pmatrix}$ $\begin{pmatrix}1\\-1\end{pmatrix}$ $\begin{pmatrix}2\\0\end{pmatrix}$	M1 A1	2	For at least two correct images For correct diagram
(v) det $\mathbf{C} = 2$	B1		State correct value
area of square has been doubled	B1	2	State correct relation a.e.f.
		9	

7.

Work done $= \frac{1}{5}mg \times 8 \quad (15.68m)$	B1	3.4
PE Loss $= \ 8mg\sin\alpha \quad (47.04m)$	B1	1.1b
KE Gain = Difference of two KE terms	M1	3.4
$= \frac{1}{2}mv^2 - \frac{1}{2}m5^2$	A1	1.1b
Work done against friction = PE Loss – KE Gain	M1	2.1
$\frac{1}{5}mg \times 8 = 8mg\sin\alpha - \left(\frac{1}{2}mv^2 - \frac{1}{2}m5^2\right)$	A1	1.1b
$v = 9.4$ or 9.37 (m s^{-1})	A1	1.1b

[7 marks]

8.

Translates problem into equations by modelling power as Fv and resistance as kv	3.3	M1	At 40 m s^{-1} Power $48000 = 40F$ Resistance $R = 40k$
Explains that at maximum speed (or when acceleration is zero) the driving force equals the resistance	2.4	E1	At maximum speed the driving force equals the resistance $R = F$
Obtains or uses a correct value for k PI by use in an expression for the resistance	1.1b	A1	$F = 1200$ N $k = 30$
Finds the resistance when travelling at 25 m s^{-1} using 'their' value of k	1.1a	M1	Resistance = 30(25) = 750
Finds the correct driving force needed when travelling with maximum power at 25 m s^{-1} May be embedded in an equation	1.1a	M1	At 25 m s^{-1} Power $48000 = 25D$ $D = 1920$
Forms a correct equation to find a	1.1b	A1	Equation of motion $1920 - 750 = 1000a$
Finds the correct value of a **stating correct units** AWRT 1.2	3.2a	A1	$a = 1.2$ m s^{-2}
Total		**7**	