L6 Further Mathematics Mock

Paper 2 (Teacher Y)

January 2023

2022-2023

Duration: **1 hour 5 minutes**

Total number of marks: **52**

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is included prior to each section of questions.

1.

In this question you must show detailed reasoning.

(a) Use an algebraic method to find the square roots of $-16 + 30\text{i}$. **[5]**

(b) By finding the cube of one of your answers to part **(a)** determine a cube root of $\frac{-99+5\text{i}}{4}$.

Give your answer in the form $a + b\text{i}$. **[2]**

2.

$$\text{f}(z) = z^3 + az + 52 \qquad \text{where } a \text{ is a real constant}$$

Given that $2 - 3\text{i}$ is a root of the equation $\text{f}(z) = 0$

(a) write down the other complex root.

(1)

(b) Hence

(i) solve completely $\text{f}(z) = 0$

(ii) determine the value of a

(4)

(c) Show all the roots of the equation $\text{f}(z) = 0$ on a single Argand diagram.

(1)

3.

(a) Express the complex number $w = 4\sqrt{3} - 4\text{i}$ in the form $r(\cos\theta + \text{i}\sin\theta)$ where $r > 0$ and $-\pi < \theta \leqslant \pi$

(4)

(b) Show, on a single Argand diagram,

(i) the point representing w

(ii) the locus of points defined by $\arg(z + 10\text{i}) = \frac{\pi}{3}$

(3)

(c) Hence determine the minimum distance of w from the locus $\arg(z + 10\text{i}) = \frac{\pi}{3}$

(3)

4.

In this question you must show detailed reasoning.

The roots of the equation $5x^3 - 3x^2 - 2x + 9 = 0$ are α, β and γ.

Find a cubic equation with integer coefficients whose roots are $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$. **[6]**

5.

In this question you must show detailed reasoning.

Two loci, C_1 and C_2, are defined as follows.

$$C_1 = \left\{z : \arg(z+2-\mathrm{i}) = \frac{1}{4}\pi\right\} \text{ and } C_2 = \left\{z : \arg(z-2-\sqrt{3}-2\mathrm{i}) = \frac{2}{3}\pi\right\}$$

By considering the representations of C_1 and C_2 on an Argand diagram, determine the locus $C_1 \cap C_2$. **[7]**

6.

The roots of the quartic equation

$$3x^4 + 5x^3 - 7x + 6 = 0$$

are α, β, γ and δ

Making your method clear and without solving the equation, determine the exact value of

(i) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$

(3)

(ii) $\dfrac{2}{\alpha} + \dfrac{2}{\beta} + \dfrac{2}{\gamma} + \dfrac{2}{\delta}$

(3)

7.

A teacher has 10 different mathematics books. Of these books, 5 are on Algebra, 3 are on Calculus and 2 are on Trigonometry.

The teacher chooses 5 of the books at random.

(a) Find the probability that 3 of the books are on Algebra. **[3]**

The teacher now arranges all 10 books in random order on a shelf.

(b) Find the probability that the Calculus books are next to each other and the Trigonometry books are next to each other. **[3]**

In this question you must show detailed reasoning.

(c) Find the probability that 2 of the Calculus books are next to each other but the third Calculus book is separated from the other 2 by at least 1 other book. **[4]**