L6 Further Mathematics Mock

Paper 1 (Teacher X)

January 2019

2018-2019

Duration: 1 hour 15 minutes (in class)

Total number of marks: 58

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is included prior to each section of questions.

The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

1.

Find the determinant of the matrix $\begin{pmatrix} a & 4 & -1 \\ 3 & a & 2 \\ a & 1 & 1 \end{pmatrix}$. **[3]**

2.

The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ -2 & a \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 7 & 3 \\ 1 & 5 \end{pmatrix}$, where $a \neq -8$ and **I** is the 2×2 identity matrix. Find

(i) $7\mathbf{A} - \mathbf{I}$, **[2]**

(ii) $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1}$. **[3]**

3.

The matrix $\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$ represents the transformation P.

(i) Describe fully the transformation P. **[3]**

Transformation Q is a stretch, parallel to the y-axis with scale factor 4.

(ii) Find the matrix that represents transformation Q. **[2]**

Transformation T is equivalent to transformation P followed by transformation Q.

(iii) Find the matrix that represents transformation T. **[2]**

(iv) Find the area of the image of the unit square under transformation T. **[2]**

4.

Find a unit vector that is perpendicular to both $\begin{pmatrix} 1 \\ 6 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix}$. Express your answer exactly. **[6]**

5.

The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5^n + 2^{n-1}$.

(i) Find u_1, u_2 and u_3. **[2]**

(ii) Hence suggest a positive integer, other than 1, which divides exactly into every term of the sequence. **[1]**

(iii) By considering $u_{n+1} + u_n$, prove by induction that your suggestion in part **(ii)** is correct. **[5]**

6.

The matrix $\mathbf{C}$ is given by $\mathbf{C} = \begin{pmatrix} a & 1 & 1 \\ 3 & a & 1 \\ 5 & 3 & 2 \end{pmatrix}$.

(i) Find the value of a for which $\mathbf{C}$ is singular. **[5]**

In the three simultaneous equations given below, p is a constant.

$$\begin{aligned} ax + y + z &= p \\ 3x + ay + z &= p - 1 \\ 5x + 3y + 2z &= p - 2 \end{aligned}$$

(ii) Write down one value of a for which these equations have a unique solution, giving a brief reason. **[1]**

(iii) Using the value of a found in **(i)**, find the value of p for which these equations are consistent. **[3]**

7.

(i) Write down a vector equation of the line through the points A (5, 1, 9) and B (8, 7, 15). **[1]**

P is the point (11, −2, 15).

(ii) Show that triangle APB is isosceles and find angle PAB. **[4]**

The point D lies on the line through A and B. Angle PAD = angle PDA.

(iii) Find the coordinates of D. **[4]**

Mechanics

8.

(i) A car of mass 800 kg is moving at a constant speed of $20\,\text{m s}^{-1}$ on a straight road down a hill inclined at an angle α to the horizontal. The engine of the car works at a constant rate of 10 kW and there is a resistance to motion of 1300 N. Show that $\sin\alpha = \frac{5}{49}$. **[4]**

(ii) The car now travels up the same hill and its engine now works at a constant rate of 20 kW. The resistance to motion remains 1300 N. The car starts from rest and its speed is $8\,\text{m s}^{-1}$ after it has travelled a distance of 22.1 m. Calculate the time taken by the car to travel this distance. **[5]**