L6 Further Mathematics Mock Teacher X 18-19 SOLUTIONS [58]

1.

1			$2a^2 + 6a - 15$	M1 M1 A1 [3]	Show correct expansion process for 3 × 3 Correct evaluation of any 2 × 2 Obtain correct answer i.s.w.	Condone sign errors for first M1 M2 for the "diagonal" method Det = $1/(2a^2 + 6a - 15)$ **only** A0

[3 marks]

2.

(i)		$7\mathbf{A} - \mathbf{I} = \begin{pmatrix} 6 & 28 \\ -14 & 7a-1 \end{pmatrix}$	B2 [2]	B1 for 3 elements correct or B1 for 4 elements correct but brackets omitted
(ii)		*Either* $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1} = \mathbf{BA}$ $\begin{pmatrix} 1 & 28+3a \\ -9 & 4+5a \end{pmatrix}$ *Or* $\mathbf{A}^{-1} = \frac{1}{a+8}\begin{pmatrix} a & -4 \\ 2 & 1 \end{pmatrix}$, $\mathbf{B}^{-1} = \frac{1}{32}\begin{pmatrix} 5 & -3 \\ -1 & 7 \end{pmatrix}$ $\begin{pmatrix} 1 & 28+3a \\ -9 & 4+5a \end{pmatrix}$	B1 M1 A1 [3] B1 M1 A1	Stated or used Attempt at multiplication of **BA** or **AB**, 2 elements correct Obtain correct answer Both correct Attempt at multiplication of their $\mathbf{A}^{-1}\mathbf{B}^{-1}$ only, ignore dets, 2 elements correct Obtain correct answer

[5 marks]

3.

(i)			B1 B1 B1 [3]	Shear, must be shear (only) otherwise 0/3 x-axis invariant, allow parallel to or along x-axis, in x direction (not in x-axis) Image of e.g. (0, 1) is (5,1) or column vectors allow 0.197^c, 11.3^o, $\tan^{-1}$ (1/5) or the complement, ignore scale factor if all OK otherwise
(ii)		$\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$	B1 B1 [2]	Each column correct
(iii)		$\begin{pmatrix} 1 & 5 \\ 0 & 4 \end{pmatrix}$	M1 A1ft [2]	Multiply matrices in correct order, or consider image of columns of P under Q Obtain correct answer, ft their (ii)
iv)		(Area =) 4	M1 A1ft [2]	Find the determinant of a relevant matrix Obtain correct answer, ft their (iii) N.B. it is possible to consider scale factor for each transformation or draw a diagram

[9 marks]

4.

Evidence of vector product employed **[M1]***

$\begin{pmatrix} 52 \\ 13 \\ 26 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ **[A1, A1, A1] oe**

Attempt to find magnitude of their $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ **[M1] dep***

Unit vector is $\frac{1}{\sqrt{21}}\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ **[A1]**

[6 marks]

5.

10	(i)		6 27 129	B1 B1 [2]	Obtain correct values Obtain 3rd correct value	
	(ii)		3	B1ft [1]	State a correct value	
	(iii)		$5^{n+1} + 2^n$	B1 M1 A1 A1 B1 [5]	Correct expression for u_{n+1} seen Attempt to factorise $u_{n+1} + u_n$ Obtain correct simplified answer Clear explanation why u_{n+1} is divisible by 3 Clear statement of induction process	Any letter, usually k or n Must deal with powers of 5 and 2 e. g. $6 \times 5^n + 3 \times 2^{n-1}$ Not $u_{n+1} + u_n$ divisible by 3 Provided other 4 marks earned

[8 marks]

6.

(i)		$2a^2 - 8a + 8$ $a = 2$	M1 M1 A1 M1 A1 **[5]**	Show correct process for det of a 3 × 3, condone sign errors (Cramer's rule is M2) Show correct processes for a 2 × 2 Obtain correct answer Attempt to solve det C = 0, must be a quadratic Obtain correct answer
(ii)		Any $a \neq 2$, so **C** non-singular or C has an inverse or det **C** $\neq 0$ or det C > 0	B1ft **[1]**	Must be consistent with their (i)
(iii)		e.g. $x + y = -1, x + y = p$ or $2p - 1 = p - 2$ $p = -1$	M1 A1 A1 **[3]**	Put $a = 2$ (their a from (i)) and attempt to eliminate x, y or z or use eqn1 +eqn2 = eqn3 Obtain a correct pair of equations e.g. in x and y or correct equation Obtain correct answer

[9 marks]

7.

i		$r = \begin{pmatrix}5\\1\\9\end{pmatrix} + \lambda\begin{pmatrix}3\\6\\6\end{pmatrix}$ oe isw	**B1** **[1]**	**NB** eg $r = \begin{pmatrix}8\\7\\15\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\2\end{pmatrix}$	**B0** for just the RHS, must see "$r =$" oe
ii		$6\times3 - 3\times6 + 6\times6 = \sqrt{6^2+(-3)^2+6^2}\times\sqrt{3^2+6^2+6^2}\cos A$ $36 = 81\cos A$ or $-36 = 81\cos A$ or better $A = 63.6°$ or 1.11 rad eg $AB = \sqrt{3^2+6^2+6^2}$ and $AP = \sqrt{6^2+(-3)^2+6^2}$ [so isosceles]	**M1** **A1** **A1** **B1** **[4]**	allow sign errors and 1 algebraic slip eg omission of power if obtuse angle found, clear explanation needed if acute angle stated as answer **NB** $AB = 9$ and $AP = 9$ stated is sufficient **B0** if answer spoiled	or $\cos A = \dfrac{9^2+9^2-(their\sqrt{90})^2}{2\times9\times9}$ $PB = 3\sqrt{10}$ $A = 63.6°$ or 1.11 rad **NB** 58.2° or $\cos\theta = \dfrac{\sqrt{10}}{6}$
iii		$\overrightarrow{PD} = \begin{pmatrix}5+3\lambda\\1+6\lambda\\9+6\lambda\end{pmatrix} - \begin{pmatrix}11\\-2\\15\end{pmatrix}$ oe $(3\lambda-6)^2+(3+6\lambda)^2+(6\lambda-6)^2 = 9^2$ oe $\lambda = \dfrac{8}{9}$[or 0] $\left(\dfrac{23}{3}, \dfrac{19}{3}, \dfrac{43}{3}\right)$ …… *Alternatively* $AD^2 = 9^2+9^2-2\times9\times9\times\cos(180-2\times63.6)$ $(3\lambda)^2+(6\lambda)^2+(6\lambda)^2 =$ their AD^2 oe $\lambda = \dfrac{8}{9}$ $\left(\dfrac{23}{3}, \dfrac{19}{3}, \dfrac{43}{3}\right)$	**M1*** **M1dep*** **A1** **A1** **[4]** …… **M1*** **M1dep*** **A1** **A1** **[4]**	 allow one algebraic slip eg omission of one power **NB** $\lambda = \dfrac{8}{3}$ if direction vector is $\begin{pmatrix}1\\2\\2\end{pmatrix}$ …… **NB** $AD = 8$ $\lambda = \dfrac{8}{3}$ if direction vector is $\begin{pmatrix}1\\2\\2\end{pmatrix}$	NB $\overrightarrow{PD} = \begin{pmatrix}3\lambda-6\\3+6\lambda\\6\lambda-6\end{pmatrix}$

iii	*Alternatively* $\overrightarrow{PE} = \begin{pmatrix} 5+3\lambda-11 \\ 1+6\lambda--2 \\ 9+6\lambda-15 \end{pmatrix}$	**M1**	E is the foot of the perpendicular from P to AB	
	$\overrightarrow{PE} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 0$	**M1**		
	$\lambda = \frac{4}{9}$	**A1**		
	$\left(\frac{23}{3}, \frac{19}{3}, \frac{43}{3}\right)$	**A1**	from $\overrightarrow{AD} = 2\overrightarrow{AE}$	
	.....	**[4]**		
	Alternatively $\overrightarrow{PD}$ found as above	**M1**	eg $\begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3\lambda \\ 6\lambda \\ 6\lambda \end{pmatrix} = \begin{pmatrix} -3\lambda \\ -6\lambda \\ -6\lambda \end{pmatrix} \cdot \begin{pmatrix} 6-3\lambda \\ -3-6\lambda \\ 6-6\lambda \end{pmatrix}$	or $\begin{pmatrix} -2 \\ -6 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 6-3\lambda \\ -3-6\lambda \\ 6-6\lambda \end{pmatrix} = 36$ or $\begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3\lambda \\ 6\lambda \\ 6\lambda \end{pmatrix} = 32$
	$\overrightarrow{AP} \cdot \overrightarrow{AD} = \overrightarrow{DA} \cdot \overrightarrow{DP}$ oe	**M1**		
	$\lambda = \frac{8}{9}$	**A1**		
	$\left(\frac{23}{3}, \frac{19}{3}, \frac{43}{3}\right)$	**A1** **[4]**		

[9 marks]

8.

5	**(i)**		Driving Force = 10000/20 (= 500)	B1	
			cv(10000/20) – 1300 + 800gsinα = 0	M1 A1	Attempt at N2L with 3 terms
			sinα = 5/49	A1 **[4]**	**AG** at least one more line of correct working (at least e.g. -800+800gsinα=*0*); allow verification (e.g. 500 – 1300 + 800 = 0)
	(ii)		800(22.1)gsinα	B1	Work done against weight; Need a value for sinα or α
			800(22.1)gsinα + 1300(22.1) + ½(800)(8^2)	M1 A1	Total work done, 3 terms needed Need a value for sinα or α; (72010 J)
			t = 3.6(0) s	M1 A1 **[5]**	Time = work done(from at least one correct energy term)/power 'Exact' is 3.6005

[9 marks]