L6 Further Mathematics Mock Teacher X 19-20 SOLUTIONS [60]

1.

(a)	$(\det(\mathbf{M}) =)\ (4)(-7)-(2)(-5)$	M1	1.1a
	$\mathbf{M}$ is non-singular because $\det(\mathbf{M}) = -18$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	$\text{Area } R = \dfrac{\text{Area } S}{(\pm)\lvert\det \mathbf{M}\rvert} = \ldots$	M1	1.2
	$\text{Area}(R) = \dfrac{63}{\lvert -18 \rvert} = \dfrac{7}{2}$ oe	A1ft	1.1b
		(2)	

2. $A^2 = \begin{pmatrix} 4 & 0 \\ 4 & 4 \end{pmatrix}$ [B1]

$hA + kI = h\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} + k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ [M1]

$\begin{pmatrix} 2h+k & 0 \\ h & 2h+k \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 4 & 4 \end{pmatrix}$ [A1]

$h = 4$ and $k = -4$ [A1, A1]

3.

Explains that det**M**=0 when **M** is singular (Seen anywhere)	AO2.4	R1	**S** is singular $\Rightarrow \begin{vmatrix} a & a & x \\ x-b & a-b & x+1 \\ x^2 & a^2 & ax \end{vmatrix} = 0$
Seeks factor by combining rows or columns to find a first linear factor for example $C_1' = C_1 - C_2$	AO3.1a	M1	$\det \mathbf{S} = \begin{vmatrix} 0 & a & x \\ x-a & a-b & x+1 \\ x^2-a^2 & a^2 & ax \end{vmatrix}$
Extracts first factor correctly	AO1.1b	A1	$= (x-a)\begin{vmatrix} 0 & a & x \\ 1 & a-b & x+1 \\ x+a & a^2 & ax \end{vmatrix}$
Combines rows or columns to find a second linear factor $R_3' = R_3 - aR_1$	AO1.1a	M1	$\det \mathbf{S} = (x-a)\begin{vmatrix} 0 & a & x \\ 1 & a-b & x+1 \\ x+a & 0 & 0 \end{vmatrix}$
Extracts second factor correctly	AO1.1b	A1	$= (x-a)(x+a)\begin{vmatrix} a & x \\ a-b & x+1 \end{vmatrix}$
Completes expansion and obtains final factor	AO1.1b	A1	$= (x-a)(x+a)(a+bx)$
Deduces correct values of x FT 'their' factors	AO2.2a	A1F	$(x-a)(x+a)(a+bx) = 0$ $x = a, -a, -\dfrac{a}{b}$
Total		**7**	

[5 marks]

4. (a) Converts L_2 to vector form: $L_2: r = \begin{pmatrix} 3 \\ 2 \\ \frac{-5}{3} \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ [B1]

Forms 2 equations and attempt to solve them [M1]

$1 - 2\lambda = 3 + 4\mu$

$3\lambda = 2 - \mu$

$3 + k\lambda = -\frac{5}{3} + 2\mu$

$\lambda = 1$ and $\mu = -1$ [A1]

Substitute into 3rd equation to find k [M1]

$k = -\frac{20}{3} = -6\frac{2}{3}$ [A1]

(b) Attempts to find vector product of the direction vectors [M1]

Equation of line is: $r = \begin{pmatrix} 7 \\ 8 \\ -10 \end{pmatrix}$ [A1]

(c) Finds scalar product of direction vectors: $2k - 11$ [M1]

Finds magnitude of both direction vectors: $\sqrt{13 + k^2}$ and $\sqrt{21}$ [M1]

Sets up the equation $\frac{2k-11}{\sqrt{21(13+k^2)}} = \frac{\sqrt{2}}{2}$ [M1]

Squares both sides and rearranges to get $26k^2 + 176k + 62 = 0$ [A1] oe

$k = -0.373$ and $k = -6.40$ [A1] both

5.

(a)	Demonstrates the rule is correct for $n = 1$ and states that it is true for $n = 1$ (may appear at any stage).	1.1b	B1	Try $n = 1$: $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^1 = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3^1 - 1 \\ 0 & 3^1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ $\therefore$ true for $n = 1$
	Multiplies $\begin{bmatrix} 1 & 3^k - 1 \\ 0 & 3^k \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ Accept any letter in place of k (condone n).	2.4	M1	Assume true for $n = k$ $\therefore \mathbf{A}^k = \begin{bmatrix} 1 & 3^k - 1 \\ 0 & 3^k \end{bmatrix}$ $\Rightarrow \mathbf{A}^k \times \mathbf{A} = \begin{bmatrix} 1 & 3^k - 1 \\ 0 & 3^k \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$
	Obtains $\begin{bmatrix} 1 & 3^{k+1} - 1 \\ 0 & 3^{k+1} \end{bmatrix}$ from multiplying $\begin{bmatrix} 1 & 3^k - 1 \\ 0 & 3^k \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ Must include an intermediate step for the top right element.	2.2a	A1	$\Rightarrow \mathbf{A}^{k+1} = \begin{bmatrix} 1 & 2 + 3(3^k - 1) \\ 0 & 3^k \times 3 \end{bmatrix}$ $= \begin{bmatrix} 1 & 3^{k+1} - 1 \\ 0 & 3^{k+1} \end{bmatrix}$
	Completes a rigorous argument and explains how their argument proves the required result. e.g. states "assume that the rule is true for $n = k$" (or equivalent) and "also true for $n = k + 1$" (or equivalent) and "for all n" and includes the base case with a conclusion. Do not accept the use of n in place of k. NMS scores 0/4	2.1	R1	$\therefore$ it is also true for $n = k + 1$ True for $n = 1$, and true for $n = k \Rightarrow$ true for $n = k + 1$ Then, by induction, it is true for all integers $n \geq 1$

(b)	Sets up two equations in (x, y) and its image (x', y')	3.1a	M1	$\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x'\\y'\end{bmatrix}$ $x' = x + 2y$ and $y' = 3y$
	Correctly substitutes $y = mx + c$ and $y' = mx' + c$	1.1b	A1	$x' = x + 2(mx + c)$ and $mx' + c = 3(mx + c)$
	Eliminates one variable to leave an equation in m, c and just one other variable.	1.1a	M1	$m(x + 2mx + 2c) + c \equiv 3mx + 3c$
	Compares coefficients to produce two correct equations in m and c	1.1b	A1	$m + 2m^2 = 3m$ and $2mc + c = 3c$
	Gives $y = 0$ or $y = x + c$ as invariant lines. Condone other incorrect invariant lines.	1.1b	B1	$m(m - 1) = 0$ and $c(m - 1) = 0$ $m = 0$ or $m = 1$ and $c = 0$ or $m = 1$
	Gives $y = 0$ and $y = x + c$ as invariant lines, with no incorrect invariant lines. NMS can score 2/6	2.2a	B1	$y = 0x + 0$ or $y = 1x + c$ Invariant lines are $y = 0$ and $y = x + c$
	Q12b: Alternative mark scheme for students who assume that all invariant lines pass through the origin – max 3 marks			
12b ALT	Sets up two equations in (x, y) and its image (x', y')	3.1a	M1	$\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x'\\y'\end{bmatrix}$ $x' = x + 2y$ and $y' = 3y$
	Eliminates three variables to leave an equation in m and just one other variable.	1.1a	M1	$x' = x + 2(mx)$ and $mx' = 3(mx)$ $m(x + 2mx) \equiv 3mx$
	Gives $y = 0$ and $y = x$ as invariant lines, with no other incorrect invariant lines. NMS can score 1/3	2.2a	B1	$m + 2m^2 = 3m$ $m(m - 1) = 0$ $m = 0$ or $m = 1$ $y = 0x$ or $y = 1x$ Invariant lines are $y = 0$ and $y = x$
(c)	Sets up at least one correct equation in x and y Accept alternative variables for this mark.	1.1a	M1	$x = x + 2y$ and $y = 3y$
	Gives $y = 0$ as the only line of invariant points. NMS can score 2/2	1.1b	A1	$y = 0$
	Total		12	

[9 marks]

7.

$T\cos 25$ $4000 = (T\cos 25)(75)$ $T = 58.8$N	B1 M1 A1 **[3]**	Seen in a WD equation WD = $F \times d$, must have component of T 58.8468…

[9 marks]

8.

(a)	Uses fact that at max speed driving force equals resistance	AO3.4	M1	$F = 30 \times 40$ $= 120$ $P = (30 \times 40) \times 40$ $= 48000$ W
	States or uses $P = Fv$	AO1.2	B1	
	Obtains correct value for power	AO1.1b	A1	
(b)	Uses resistance model in a three term equation of motion.	AO3.4	M1	$F - 30 \times 25 = 1200a$ $F = 1200a + 750$ $48000 = 25(1200a + 750)$ $a = \frac{1920 - 750}{1200}$ $= 0.975 \text{ m s}^{-2}$ $= 0.98 \text{ m s}^{-2}$ to 2 sf
	Obtains a correct equation of motion.	AO1.1b	A1	
	Solves 'their' equation of motion for a.	AO1.1a	M1	
	Obtains correct acceleration. FT 'their' equation provided both M1 marks awarded	AO1.1b	A1F	
	Total		**7**	

9.

6	**(i)**	$4(8) + 3(-10) = 4v_A + 3v_B$	M1* A1	Attempt at use of conservation of momentum
		$\frac{1}{2}(4)(8)^2 + \frac{1}{2}(3)(10)^2 - \frac{1}{2}(4)v_A^2 - \frac{1}{2}(3)v_B^2 = 121.5$	M1* A1	Attempt at use of KE(before) – KE (after) = 121.5
			M1 dep*	Obtaining quadratic equation in either v_A or v_B $\left(7v_B^2 - 4v_B - 416 = 0, \quad 28v_A^2 - 16v_A - 935 = 0\right)$ and attempt to solve quadratic for either v_A or v_B
		$v_A = -5.5$ $(v_A = 6.0714...)$ so speed of A is 5.5 (m s^{-1})	A1	cao; must be positive
		$v_B = 8$ $(v_B = -7.428...)$ so speed of B is 8 (m s^{-1})	A1	cao; must be positive
		Both particles are moving in the reverse direction to their original motion	A1 **[8]**	Or an equivalent statement consistent with their v_A and v_B; left and right not sufficient without a diagram; moving away from each other needs a diagram also
	(ii)	$v_A - v_B = -e(8 - (-10))$ $e = 0.75$	M1 A1 **[2]**	Attempt at use of coefficient of restitution, right way round, v_A and v_B substituted

[9 marks]