

L6 Further Mathematics Mock

Paper 1 (Teacher X)

January 2023

2022-2023

Duration: 1 hour 7 minutes

Total number of marks: 53

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is included prior to each section of questions.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

1.

The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix}$$

(a) Find $\det \mathbf{A}$

[1 mark]

(b) Find \mathbf{A}^{-1}

[1 mark]

(c) Given that $\mathbf{AB} = \begin{bmatrix} 9 & 6 \\ 5 & 12 \end{bmatrix}$ and $\mathbf{M} = 2\mathbf{A} + \mathbf{B}$ find the matrix \mathbf{M}

[3 marks]

2.

(a) Determine whether the point $(19, -12, 17)$ lies on the line $\mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$. [3]

Vectors \mathbf{a} and \mathbf{b} are given by $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix}$.

(b) (i) Find, in degrees, the angle between \mathbf{a} and \mathbf{b} .

[3]

(ii) Find a vector which is perpendicular to both \mathbf{a} and \mathbf{b} .

[2]

3.

Prove that $3^n > 10n$ for all integers $n \geq 4$.

[5]

4.

The lines l_1 and l_2 have equations

$$l_1 : \mathbf{r} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$$

$$l_2 : \mathbf{r} = \begin{bmatrix} -12 \\ a \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

(a)

Given that the lines l_1 and l_2 intersect, calculate the value of the constant a

[4 marks]

(b)

Hence, find the coordinates of the point of intersection of l_1 and l_2

[1 mark]

5.

The point P has coordinates $(8, 3, 2)$.

The point Q is the image of P under the transformation reflection in the plane $y = 0$.

(a) Write down the coordinates of Q . (1)

The point R is the image of P under the transformation rotation through 120° anticlockwise about the y -axis.

(b) Determine the exact coordinates of R . (2)

(c) Hence find $|\overrightarrow{PR}|$ giving your answer as a simplified surd. (2)

(d) Show that \overrightarrow{PR} and \overrightarrow{PQ} are perpendicular. (1)

(e) Hence determine the exact area of triangle PQR , giving your answer as a surd in simplest form. (2)

6. Let the matrix A be given by $\begin{pmatrix} a & 1 \\ -1 & 3 \end{pmatrix}$ where a is a constant

(i) Given that the determinant of A is 25 find the value of a . [2]

(ii) You are given instead that the following system of equations does **not** have a unique solution.

$$ax + y = -2$$

$$-x + 3y = -6$$

Determine the value of a . [2]

7.

The matrix A is given by $A = \frac{1}{13} \begin{pmatrix} 5 & 12 \\ 12 & -5 \end{pmatrix}$.

You are given that A represents the transformation T which is a reflection in a certain straight line. You are also given that this straight line, the mirror line, passes through the origin, O .

- (a) Explain why there must be a line of invariant points for T . State the geometric significance of this line. [2]
- (b) By considering the line of invariant points for T , determine the equation of the mirror line. Give your answer in the form $y = mx + c$. [4]

The coordinates of the point P are $(1, 5)$.

- (c) By considering the image of P under the transformation T , or otherwise, determine the coordinates of the point on the mirror line which is closest to P . [3]
- (d) The line with equation $y = ax + 2$ is an invariant line for T . Determine the value of a . [2]

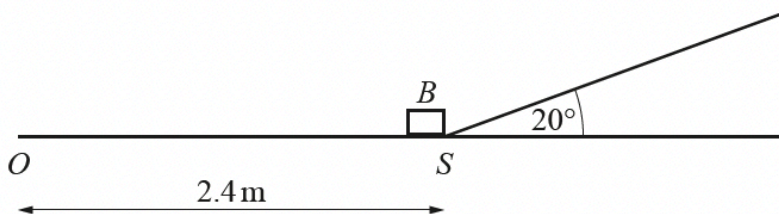
8.

A small box B of mass 4.2 kg is initially at rest at a point O on rough horizontal ground. A horizontal force of magnitude 35 N is applied to B .

B moves in a straight line until it reaches the point S which is 2.4 m from O . At the instant that B reaches S its speed is 4.5 ms^{-1} .

- (a) (i) Find the energy lost due to the resistive forces acting on B as it moves from O to S . [3]
- (ii) Deduce the magnitude of the average resistive force acting on B as it moves from O to S . [2]

When B reaches S , the force is no longer applied. B continues to move directly up a smooth slope which is inclined at 20° above the horizontal (see diagram).



- (b) (i) State an assumption required to model the motion of B up the slope with only the information given. [1]
- (ii) Using the assumption made in part (b)(i), determine the distance travelled by B up the slope until the instant when it comes to rest. [3]