L6 Further Maths Teacher Y Mock SOLUTIONS [51]

1.

2	**(i)**	$x = 2\sqrt{3}\cos(-\frac{\pi}{3}), y = 2\sqrt{3}\sin(-\frac{\pi}{3}), x^2 + y^2 = 12, y = -x\sqrt{3}$ $\sqrt{3} - 3\,\mathrm{i}$	M1 A1 **[2]**	Correct trig expression for x or y allow a positive angle or 2 equations for x and y, not involving trig Obtain correct answer as a complex number, extra answers not rejected gets A0
	(ii)	$\sqrt{3} + 3\mathrm{i}$ $-1 - (4\sqrt{3})\mathrm{i}$ $-\frac{1}{49} + \frac{4\sqrt{3}}{49}\mathrm{i}$	B1ft M1 A1 M1 A1 **[5]**	Correct conjugate seen, ft from their z in (i) Expand denominator Correct value seen Attempt to rationalise Obtain correct answer a.e.e.f. **N.B.** if 2 answers given in (i) award marks for better solution in (ii)

[7 marks]

2.

5	**(i)**		$2u^3 + 12u^2 + 27u + 25 = 0$	M1 A2 **[3]**	Substitute and attempt to simplify Obtain correct equation, A1 for only 1 error	Missing = 0 is an error
	(ii)		*Either* $\frac{\sum\alpha'\beta'}{\alpha'\beta'\gamma'}$ $-\frac{27}{25}$ *Or* $25y^3 + 27y^2 + 12y + 2 = 0$ *Or* $\frac{\sum\alpha'\beta'}{\alpha'\beta'\gamma'}$	M1 A1 M1 A1ft **[4]** M1 A1ft M1 A1ft M1 A1 M1 A1	Combine 3 terms with correct denominator Obtain correct expression in their notation Attempt to use values from **(i)** correctly Obtain correct answer **with no errors seen** Use substitution $y = \frac{1}{u}$ Obtain correct cubic equation, from their (i) Use correct symmetric function Obtain correct answer Combine 3 terms with correct denominator Obtain correct expression in their notation Expand numerator and denominator and use values from original equation correctly Obtain correct answer **with no errors seen**	Must be ±c/**a and** ±d/**a** for M1 ft for their answer in (i) Condone ±, but must be "/2"

[7 marks]

3.

6.	**(i)**	$3 + 3\mathrm{i}$ $\sqrt{5}$ $\lvert z - 3 - 3\mathrm{i}\rvert = \sqrt{5}$	B1 B1 M1 A1ft **[4]**	Obtain centre as a complex number, allow (3, 3) but not (3, 3i), give if correct in locus equation Obtain correct radius a.e.f. condone decimals Use correct form for locus Obtain correct answer from their centre and radius, $z = x + \mathrm{i}y$ is acceptable
	(ii)	Circle, centre in 1st quadrant, not touching or intersecting axes Straight line with –ve slope through (3, 3)	B1 B1 **[2]**	Centre need not be (3, 3) (3, 3) may be implied by working from (i) must be longer than the diameter. N.B. No circle drawn, B1 for l may be earned
	(iii)	$2 + 5\mathrm{i}$ $4 + \mathrm{i}$	M1 A1 A1 **[3]**	Gradient of l is – 2 used or attempt to solve Cartesian equations for C and l M0 if equations for C and/or l contain i. Obtain correct answers, must be complex numbers

[9 marks]

4.

7	(i)			M1	Attempt to equate real and imaginary parts of $(x+iy)$ and $5+12i$	
			$x^2-y^2=5, 2xyi=12i$	A1	Obtain both results or equivalent	
				M1	Obtain and solve a quadratic in x^2 or y^2 or solve by inspection	
			$3+2i$ and $-3-2i$ or $\pm(3+2i)$	A1 A1	Obtain correct answers as complex numbers	
				[5]	**S.C. $\pm(3\pm 2i)$ scores A1**	
7	(ii)			M1	Solve using quadratic formula or complete square	
			$(4\pm 2\sqrt{5+12i})/2$	A1	Obtain correct answers, or simpler version	
				M1	Use result(s) from **(i)**	
			$5+2i$ and $-1-2i$ or $2\pm(3+2i)$	A1 A1	Obtain correct answers	If more than 2 roots A0 A0
				[5]		

[10 marks]

5.

9	(i)		*Either*	M1	Substitute α into equation and rearrange	$\alpha^3-k\alpha^2-2=0$
			$k=\alpha-\frac{2}{\alpha^2}$	A1	Obtain **given** answer a.e.f.	
				[2]		
			Or	M1	Substitute for k and x in terms of α and simplify	
				A1	Show simplification leads to consistency	e.g. "LHS = 0"
			Or	M1	Eliminate β and γ from symmetric functions	Don't penalise sign errors
				A1	Obtain **given** answer correctly	
	(ii)			B1	State or use $(\gamma)=u-iv$	
				M1*	Use sum of roots $=(\pm)k$ (Can use $\sum\alpha\beta$ with $\alpha\beta\gamma$)	$\alpha+u+iv+u-iv=-(-k)$
			$u=-\frac{2}{2\alpha^2}$ or better	DM1	Rearrange to get u	$\alpha+2u-\alpha-\frac{2}{\alpha^2}$
				A1	Obtain correct answer	
				[4]		

[10 marks]

6.

8	(i)		${}^5C_2$ oe seen anywhere or num= 10 alone	M1	$\frac{1}{8}\times\frac{1}{7}\times\frac{5}{6}\times\frac{4}{5}$ or $\frac{20}{1680}$ or $\frac{1}{84}$ oe seen	alone or ×... eg $\frac{2}{8}\times\frac{1}{7}\times\frac{5}{6}\times\frac{4}{5}$ M1
			$\frac{{}^5C_2}{{}^8C_4}$ oe or $\frac{{}^5C_2\times 4!}{{}^8P_4}$ oe all correct	M1	$\frac{1}{8}\times\frac{1}{7}\times\frac{5}{6}\times\frac{4}{5}\times{}^4C_2\times 2$ or $\frac{1}{8}\times\frac{1}{7}\times\frac{5}{6}\times\frac{4}{5}\times 4!\div 2$ oe or $\frac{1}{8}\times\frac{1}{7}\times\frac{5}{6}\times\frac{4}{5}\times 12$ oe all correct	$\frac{4}{8}\times\frac{3}{7}\times\frac{4}{6}$ oe all correct M2 NB $\frac{\text{incorrect}}{{}^8C_4}$ does not score
			$=\frac{1}{7}$ or 0.143 (3 sf)	A1	Correct ans scores M1M1A1 regardless of method.	
				[3]		
8	(ii)		6! × 2 alone or 5! × 6 × 2 alone oe	M2	M1 for 6! or 5! × 6 or ${}^5P_5$ or 720 seen NB 5! scores M0 unless 5! × 6 or 5! × 12	M1 for 7! × 2 alone NB 7! scores M0 unless 7! × 2 alone
			= 1440	A1		
				[3]		
8	(iii)		6! × 4 alone or 6! × 2 × 2 alone	M2	M1 for 6! or ${}^5P_5$ or 720 seen or 5! × 6 seen but NOT from 5!×3!	5!: M0 unless 5!×6 or 5!×12 or 5!×24
			= 2880	A1		
				[3]		

[9 marks]