

# **L6 Further Mathematics Mock**

## **Paper 2 (Teacher Y)**

**January 2020**

**2019-2020**

**Duration: 1 hour (in class)**

**Total number of marks: 46**

*Write your answers on file paper.*

**You are permitted to use a scientific or graphical calculator in this paper.**

**Final answers should be given to a degree of accuracy appropriate to the context.**

**Relevant information from the formula booklet is included prior to each section of questions.**

## Pure (34 marks)

1.

The equation  $x^3 - 8x^2 + cx + d = 0$  where  $c$  and  $d$  are real numbers, has roots  $\alpha, \beta, \gamma$ .

When plotted on an Argand diagram, the triangle with vertices at  $\alpha, \beta, \gamma$  has an area of 8.

Given  $\alpha = 2$ , find the values of  $c$  and  $d$ .

Fully justify your solution.

[5 marks]

2.

The complex number  $a + ib$  is denoted by  $z$  and the complex number  $c + id$  is denoted by  $w$ .

It is given that  $z^2 = z*w$ .

(i) Show that  $2ab = ad - bc$ .

[4]

(ii) Given that the real part of  $w = 0$ , find the values of  $b$  in terms of  $a$ .

[6]

[10 marks]

3.

(a) Shade on an Argand diagram the set of points

$$\left\{ z \in \mathbb{C} : |z - 4i| \leq 3 \right\} \cap \left\{ z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z + 3 - 4i) \leq \frac{\pi}{4} \right\}$$

(6)

The complex number  $w$  satisfies

$$|w - 4i| = 3$$

(b) Find the maximum value of  $\arg w$  in the interval  $(-\pi, \pi]$ .

Give your answer in radians correct to 2 decimal places.

(2)

[8 marks]

4.

(i) Use an algebraic method to find the square roots of the complex number  $9 + 40i$ .

[6]

(ii) Show that  $9 + 40i$  is a root of the quadratic equation  $z^2 - 18z + 1681 = 0$ .

[1]

(iii) By using the substitution  $z = \frac{1}{u^2}$ , find the roots of the equation  $1681u^4 - 18u^2 + 1 = 0$ . Give your answers in the form  $x + iy$ , where  $x$  and  $y$  are real.

[4]

[11 marks]

## Statistics (12 marks)

5.

The discrete random variable  $R$  has the following probability distribution.

$r$	-2	0	$a$	4
$P(R = r)$	0.3	$b$	$c$	0.1

It is known that  $E(R) = 0.2$  and  $\text{Var}(R) = 3.56$

Find the values of  $a$ ,  $b$  and  $c$ .

**[4 marks]**

6.

- (i) The seven digits 1, 1, 2, 3, 4, 5, 6 are arranged in a random order in a line. Find the probability that they form the number 1452163. [3]
- (ii) Three of the seven digits 1, 1, 2, 3, 4, 5, 6 are chosen at random, without regard to order.
- (a) How many possible groups of three digits contain two 1s? [1]
- (b) How many possible groups of three digits contain exactly one 1? [2]
- (c) How many possible groups of three digits can be formed altogether? [2]

**[8 marks]**