

L6 Further Maths Teacher Y Mock SOLUTIONS [46]

1.

Writes β and γ in the form $p \pm qi$ (seen anywhere in the solution)	AO2.5	B1	Real coefficients $\Rightarrow \beta = p + qi$ and $\gamma = p - qi$
Uses "sum of the roots = $-b/a$ " together with a conjugate pair to determine the real part (p) of β and γ	AO3.1a	M1	$\alpha + \beta + \gamma = 8$ $\Rightarrow 2 + p + qi + p - qi = 8$ $\Rightarrow 2 + 2p = 8$ $\Rightarrow p = 3$
Uses '(their p)' -2 and the area of the triangle on an Argand diagram to determine the imaginary parts of β and γ	AO3.1a	M1	$(p - 2)q = 8$ $\Rightarrow q = 8$
Uses a correct method to find the value of c or d using 'their' values of $p \pm qi$	AO1.1a	M1	$\beta = 3 + 8i$ and $\gamma = 3 - 8i$
Obtains correct values for c and d . CAO	AO1.1b	A1	$d = -\alpha\beta\gamma = -146$ $c = \sum \alpha\beta = 85$
Total		5	

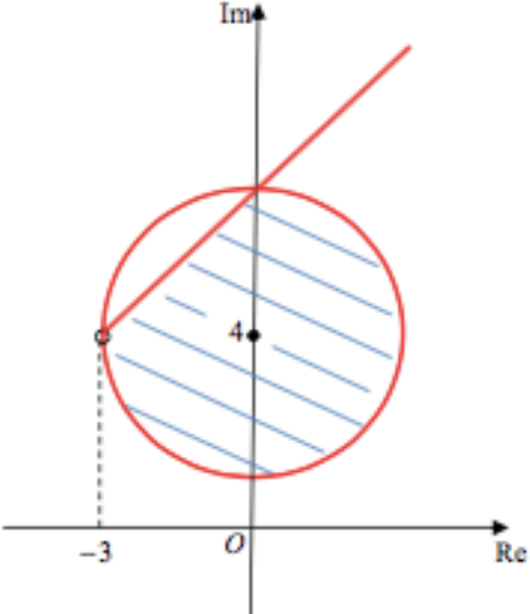
[5 marks]

2.

(i)	$2ab = ad - bc$	B1 M1 M1 A1 [4]	State or use $z^* = a - ib$ Attempt to expand both expressions Equate imaginary parts Obtain given answer N.B. ignore errors in real parts
(ii)	Either $a^2 - b^2 = bd, ad = 2ab$ $b = \pm \frac{a}{\sqrt{3}}$ Or $a^2 - b^2 + 2abi = adi + bd$ $d = \frac{(a^2 - b^2 + 2abi)(b - ia)}{a^2 + b^2}$ $b = \pm \frac{a}{\sqrt{3}}$	B1 M1 A1 M1 A2 [6] B1 M1 A1 M1 A2	State or use $c = 0$ Equate real parts, c need not = 0 at this stage Obtain 2 correct equations a.e.f. Eliminate d Obtain correct answers a.e.f., A1 for 1 correct answer S.C. $a = \pm\sqrt{3}b$ gets A1 State or use $c = 0$ in given expression Rearrange to make d the subject and rationalise Obtain correct answer Equate imaginary part to 0 Obtain correct answers a.e.f., A1 for 1 correct answer S.C. $a = \pm\sqrt{3}b$ gets A1

[10 marks]

3.

Question	Scheme	Marks	AOs
8(a)		M1	1.1b
		A1	1.1b
		M1	1.1b
		A1	2.2a
		M1	3.1a
		A1	1.1b
		(6)	
(b)	$(\arg w)_{\max} = \frac{\pi}{2} + \arcsin\left(\frac{3}{4}\right)$	M1	3.1a
	$= 2.42 \text{ (2 dp) cao}$	A1	1.1b
	(2)		
(8 marks)			
Notes:			
<p>(a)</p> <p>M1: Circle</p> <p>A1: Centre (0, 4) and above the real axis</p> <p>M1: Half-line</p> <p>A1: (-3, 4) positioned correctly and the half-line intersects the top of the circle on the y-axis</p> <p>M1: Depends on both previous M marks Shades in a region inside the circle and below the half-line</p> <p>A1: cso</p> <p>Note: Final A1 mark is dependent on all previous marks being scored in part (a)</p>			
<p>(b)</p> <p>M1: Uses trigonometry to give an expression for an angle in the range $\left(\frac{\pi}{2}, \pi\right)$ or $(90^\circ, 180^\circ)$</p> <p>A1: 2.42 cao</p>			

[8 marks]

4.

10.	(i)	$x^2 - y^2 = 9, \quad xy = 20$ $\pm(5 + 4i)$	M1 A1 M1 M1 A1 A1 [6]	Equate real and imaginary parts of $9 + 40i$ and $(x + iy)^2$ Obtain correct equations a.e.f., e.g. allow $2ixy = 40i$ Rearrange to obtain quadratic in x^2 or y^2 Solve quadratic in x^2 or y^2 and square root Obtain one correct answer as a complex number Obtain other answer as complex number a.e.f. $\pm 5 \pm 4i$ is OK for A2, $\pm(5 \pm 4i)$ scores A1 only
	(ii)		B1 [1]	Use sum and product of roots or solve equation or substitute into equation
	(iii)	$u = \frac{1}{5 + 4i}$ $\pm(\frac{5}{41} + \frac{4}{41}i), \pm(\frac{5}{41} - \frac{4}{41}i)$	B1 M1 A1 A1 [4]	Use substitution to obtain one of the equations from other equation , (must have = 0) Use result from (i) (either root) Obtain any 2 correct answers Obtain other 2 correct answers S.C.: If they solve quadratic for u^2 directly, use scheme above, M1 when they have used (i) to get a value for u .

[11 marks]

5.

Uses sum of probs = 1	AO1.2	M1	$0.4 + b + c = 1$ $b + c = 0.6$
Uses formula for $E(R)$	AO1.1a	M1	$E(R) = 0.2 \Rightarrow$ $(-2 \times 0.3) + (0 \times b) + (a \times c) + (4 \times 0.1) = 0.2$ $ac = 0.4$
Uses formula for variance $E(X^2) - (E(X))^2$	AO1.1a	M1	$E(X^2) - (E(X))^2 = (4 \times 0.3) + (0 \times b)$ $+ (a^2 \times c) + (16 \times 0.1) - (0.2)^2 = 3.56$ $a^2c = 0.8$
Obtains a, b and c CAO	AO1.1b	A1	From (2) and (3) $a = 2$ Hence $c = 0.2$ and $b = 0.4$
Total		4	

[4 marks]

6.

6	(i)	$7!$ or 5040 or 7P_7 seen $1 + \frac{7!}{2} \quad \text{or} \quad \frac{2}{7!}$ $= \frac{1}{2520}$ or 0.000397 (3 sf)	M1 M1 A1 [3]	or $5! \times ({}^6C_2 + 6)$ NOT $5! \times {}^6C_2$ $\frac{1}{5 \times (6C_2 + 6)}$ or $\frac{2}{5040}$ oe	or $\frac{2}{7} \times \frac{1}{6} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2}$ alone M2 or ≥ 5 correct fracs mult: or 6 correct fracs mult $\times \dots$ M1
6	(ii)	(a)	5	B1 [1]	Ignore any working seen
6	(ii)	(b)	5C_2 alone (or $\times {}^2C_2$) or ${}^6C_3 + 2(1)$ or $\frac{2}{7} \times {}^7C_3$ or ${}^5P_2 + 2$ $= 10$	M1 A1 [2]	alone, eg NOT ${}^5C_2 \times \dots$ or ${}^5C_2 + \dots$ But allow 5C_2 as denom of prob M1A0
6	(ii)	(c)	$"5" + "10" + {}^5C_3$ $= 25$	M1 A1f [2]	or ${}^6C_3 + "5"$ or ${}^7C_3 - "10"$ or ${}^7C_3 - {}^5C_2$ ft (a) &/or (b) only if working seen Allow as denom of a prob M1A0

[8 marks]