Acceleration in horizontal circular motion

Starter

1. **(Review of last lesson)**
   A cyclist completes a circuit of a circular track in 14 s. Calculate her exact angular speed.

Notes

The direction of the velocity is along a tangent to the circle. When the word velocity is used on its own, you can assume it refers to the linear, or ***tangential***, speed.

Movement in a circle has two types of acceleration — tangential, a_t and radial, a_r — which are perpendicular to each other. As the names suggest, ***tangential acceleration*** acts along the tangents to the circle and ***radial acceleration*** acts along the radius of the circle.

We will consider particles moving at a constant velocity around a horizontal circle — in which case there is no tangential acceleration i.e. $a_t = 0$.

The direction of the ***acceleration is towards the centre of the circle***. Since $F = ma$, the force is in the same direction as the acceleration so the ***force is towards the centre of the circle***. This radial force (i.e. along the radius) is called the ***centripetal force***.

What is the formula for radial acceleration, a_r?

The proof of the following formula is beyond AS and is not needed for the course:

$$a_r = r\omega^2$$

Using $v = r\omega$, we can substitute r: $\qquad a_r = v\omega$

Using $\omega = \dfrac{v}{r}$, we can substitute ω: $\qquad a_r = \dfrac{v^2}{r}$

There are three formulae for the radial acceleration — the key is to chose the most appropriate one for the question.

Remember: a_r is directed towards the centre of the circle.

A particle travelling around a circle is always accelerating towards the centre of the circle, but without getting any closer.

E.g. 1 A turntable is rotating at a constant rate of 45 rpm. A fly is standing on it, 8 cm from its centre. Find:

(a) the angular speed

(b) the tangential speed and

(c) the acceleration of the fly, stating the direction of the acceleration.

Working: (a) Angular speed, $\omega = \dfrac{45}{60} \times 2\pi = \dfrac{3\pi}{2}$ rad/s

Problems involving the centripetal force

By using the radial acceleration, a_r, and $F = ma$, problems can be solved involving forces. The key is to select the correct equation from $a_r = r\omega^2$ or $a_r = v\omega$ or $a_r = \frac{v^2}{r}$.

E.g. 2 A particle of mass 0.4 kg moves on a horizontal circle, centre O. The speed of the particle is 3 m/s and the force on P directed towards O is 15 N. Calculate the distance OP.

Working: Using $F = ma$ radially: $15 = 0.4a_r \Rightarrow a_r = \frac{15}{0.44} = 37.5$

Using $a_r = \frac{v^2}{r}$: $37.5 = \frac{3^2}{r} \Rightarrow r = \frac{9}{37.5} = \frac{6}{25} = 0.24$

The distance OP is 0.24 m.

E.g. 3 A radial force of 20 N is required to maintain a particle moving in a horizontal circle of diameter 1.8 m with speed 4.8 m/s. Calculate the mass of the particle.

E.g. 4 A particle P of mass 0.3 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a fixed point O on a smooth horizontal surface. P moves in a circular path with speed 4 m/s.

(a) Calculate the tension in the string.

Given that the tension in the string cannot exceed 30 N, find

(b) the maximum tangential speed of P in m/s, and

(c) the maximum angular speed in rad/s.

Video: Acceleration in horizontal circle

Solutions to Starter and E.g.s

Exercise

p87 4B Qu 1-6 (AS), 7-8 (A2 - needs coefficient of friction)

Summary

Radial acceleration, a_r, is directed towards the centre of the circle and is given by:

$$a_r = r\omega^2 \quad \ldots\text{or}\ldots \quad a_r = v\omega \quad \ldots\text{or}\ldots \quad a_r = \frac{v^2}{r}$$

By using the radial acceleration, a_r, and $F = ma$, problems can be solved involving forces. The key is to select the correct equation for radial acceleration, a_r.