Average and Spread of Discrete Random Variables

Starter

1. **(Review of last lesson)**
   The discrete random variable R has probability distribution function given by
   $P(R = r) = c(3 - r)$ for $r = 0, 1, 2, 3$. Find the value of the constant c.

2. In the previous lesson we found the probability distribution, X, of a biased 4-sided dice was:

x:	1	2	3	4
$P(X = x)$:	0.48	0.24	0.16	0.12

   (a) If we rolled the dice 100 times, how many of each score would we expect to get?
   (b) Using your expected frequencies from (a), find the mean score per roll.

Notes

Expected value, or mean, of a random variable

We can calculate the expected value from the probability distribution.

E.g. 1 How could the answer to 2(b) be found from the probability distribution?

x:	1	2	3	4
$P(X = x)$:	0.48	0.24	0.16	0.12

Working:

x:	1	2	3	4
$P(X = x)$:	0.48	0.24	0.16	0.12

Mean $= 1 \times 0.48 + 2 \times 0.24 + 3 \times 0.16 + 4 \times 0.12 = 1.92$

The value 1.92 is called the mean of X or expected value of X.

Notation

The ***expected value***, or **mean**, of X is usually written $E(X)$ or it is given the Greek letter μ (mu).

In general for a discrete random variable X:

$$\mu = E(X) = \sum xP(X = x)$$

E.g. 2 A random variable X has a pdf defined as shown. Find $E(X)$.

x:	-2	-1	0	1	2
$P(X = x)$:	0.3	0.1	0.15	0.4	0.05

Variance and standard deviation

Population variance is defined as $\dfrac{\sum (x-\mu)^2 f}{\sum f} = \dfrac{\sum x^2 f}{\sum f} - \mu^2$

For a random variable, X, the ***variance*** is found by:

$$\begin{aligned} \text{Var}(X) &= \sum (x-\mu)^2 P(X = x) \\ &= \sum x^2 P(X = x) - \mu^2 \qquad \text{where } \mu = E(X) \\ &= E(X^2) - E^2(X) \end{aligned}$$

In general, $\text{Var}(X) = E(X^2) - E^2(X)$

Standard deviation, σ ("sigma") $\sigma = \sqrt{\text{Var}(X)}$

N.B. $E(X) = \sum xP(X=x)$
$E(X^2) = \sum x^2P(X=x)$
$E(X^2) \neq E^2(X)$
$E^2(X)$ means we squared the mean i.e. $\left[E(X)\right]^2$
$E(X^2)$ means we squared all the $x-$values and then calculated the expectation

E.g. 3 The random variable X has probability distribution as shown in the table.

x	1	2	3	4	5
$P(X=x)$	0.1	0.3	0.2	0.3	0.1

Find Var(X) and hence the standard deviation.

Working
$\mu = E(X) = (1\times0.1)+(2\times0.3)+(3\times0.2)+(4\times0.3)+(5\times0.1)$
$E(X) = 3$ ***we could get this from the symmetry of the distribution***

$E(X^2) = (1^2\times0.1)+(2^2\times0.3)+(3^2\times0.2)+(4^2\times0.3)+(5^2\times0.1)$
$= 10.4$

$\text{Var}(X) = E(X^2) - E^2(X) = 10.4 - 3^2 = 1.4$

Standard deviation $= \sqrt{1.4} = \dfrac{\sqrt{35}}{5} \approx 1.18$

E.g. 4 Two fair cubical dice are rolled and S is the sum of their scores. Find:
(a) the distribution of S.
(b) the expected value of S
(c) the standard deviation of S

Video: Expected values E(X)
Video: Variance Var(X)

Solutions to Starter and E.g.s

Exercise
p24 2A Qu 1i, 2-9, (10 red)

Summary
The ***expected value***, or **mean**, of X: $\mu = E(X) = \sum xP(X=x)$
Variance: $\text{Var}(X) = E(X^2) - E^2(X)$
Standard deviation: $\sigma = \sqrt{\text{Var}(X)}$

$E(X^2) = \sum x^2P(X=x)$
$E(X^2) \neq E^2(X)$
$E^2(X)$ means we squared the mean i.e. $\left[E(X)\right]^2$
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