# Binomial Distribution (AS Ma)

**Starter**

1. **(Review of last lesson)** The discrete uniform random variable $X$ can take the values $22, 26, 30, \ldots, 146$. Find $E(X)$ and $\text{Var}(X)$.

2. An unbiased coin is flipped 4 times and the outcomes "Head" ($H$) and "not a Head" ($H'$) are noted.
   (a) Write down the possible outcomes.
   (b) Write down the probability of getting each outcome.
   (c) Let $X$ be the discrete random variable "number of Heads obtained". Copy and complete the table:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | | | | | |

**Notes**

Question 2 of the starter is an example of a binomial distribution and its theory allows us to solve problems without having to write down all the possible outcomes.

This is another area that will be covered in greater depth in the AS Maths course, but the next lesson cannot be done without prior understanding of the topic.

***Requirements for a distribution to be binomially distributed***

- There are a fixed number of trials, $n$.
- Each trial has only $2$ outcomes ("success" and "failure").
- The probability of success, $p$, does not change from trial to trial.
- The outcome of each trial is independent of the outcome of the other trials.

***E.g. 1*** Decide whether the following are binomial distributions.
(a) A dice is rolled 10 times and the number of $6$s obtained is counted.
(b) A coin is flipped until a tail comes up.
(c) The number of penalties scored in a shoot-out by a team to decide the result of a football game.

***Binomial formula***

Returning to the example in the starter there were 4 trials and the probability of success (i.e. getting head) was $\frac{1}{2}$.

If we consider rolling three heads then we have $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^1$

$\frac{1}{2}$ is raised to the power of $3$ because we get three heads.

$\frac{1}{2}$ is raised to the power of 1 because we get one non-head.

This gives us $\frac{1}{16}$ so where does the $\times 4$ comes from to get $\frac{4}{16}$?

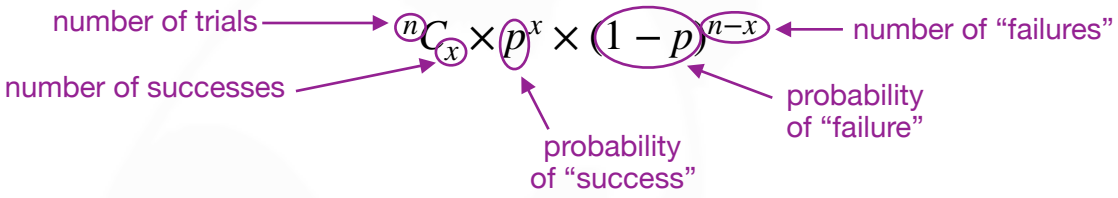
It comes from the binomial coefficients (hence the name).

Expanding $(x+y)^4$, gives coefficients: 1 4 6 4 1

i.e. ${}^4C_0$ ${}^4C_1$ ${}^4C_2$ ${}^4C_3$ ${}^4C_4$

In general, if $X \sim \text{B}(n, p)$ then $P(X = x) = {}^nC_x \times p^x \times (1-p)^{n-x}$

where $n$ is the number of trials
$p$ is the probability of "success"
$x$ is the number of "successes"
$n - x$ is the number of "failures"
$1 - p$ is the probability of "failure"




***E.g. 2*** An unbiased 6-sided dice is rolled 7 times. Let $X$ be the number of 6s obtained. Find
(a) the probability of getting two 6s i.e. $P(X = 2)$
(b) $P(X \leq 2)$

***Working:*** (a) $P(X=2) = {}^7C_2 \times \frac{1}{6}^2 \times \frac{5}{6}^5 = 0.2344$

***Calculating binomial probabilities on the Classwiz***
Menu >> 7: Distribution
At this point there are two options you could choose:
- 4: Binomial PD — for $P(X = x)$ type questions
- Down arrow >> 1: Binomial CD — for $P(X \leq x)$ type questions >> 2: Variable

***N.B.*** For $P(X < x)$, do $P(X \leq x - 1)$
Press AC to go back to the input screen
To store values press STO >> A
PD ≡ probability distribution
CD ≡ cumulative distribution

***E.g. 3*** Let $Y \sim \text{B}(6, 0.25)$. Using your calculator, find:
(a) $P(Y = 3)$ (b) $P(Y < 3)$ (c) $P(Y > 3)$

***Working:*** (a) $P(Y = 3) = 0.1318$ ***use Binomial PD***

Video: Binomial distribution
Video: Binomial probabilities on calculators
Video: Cumulative probability tables
Video: Calculating binomial probabilities

Solutions to Starter and E.g.s

**Exercise**

1. (Non-Calc) The random variable $X \sim \text{B}\left(8\ ,\ \frac{1}{3}\right)$, find:
   (a) $P(X = 2)$ (b) $P(X \leq 2)$ (c) $P(X \geq 2)$

2. (Calc) The random variable $T \sim \text{B}(12,\ 0.6)$, find:
   (a) $P(T = 6)$ (b) $P(T > 7)$ (c) $P(6 \leq T \leq 9)$

3. (Non-Calc) A balloon manufacturer claims that $95\ \%$ of balloons will not burst when blown up. Twenty balloons are blow up for a birthday party:
   (a) What is the probability that none of them will burst?
   (b) Find the probability that exactly two burst.
   (c) There are 17 children coming to the party. What is the probability there are enough balloons?

4. (Non-Calc) Team A has probability $\frac{2}{3}$ of winning whenever it plays. Given that A plays $4$ games, find the probability A wins more than half the games.

5. (Calc) The random variable $U \sim \text{B}(16,\ 0.2)$, find:
   (a) $P(U = 4)$ (b) $P(U < 4)$ (c) $P(U > 4)$

**Answers to Exercise**

1. (a) $0.2731$ (b) $0.4682$ (c) $0.8049$

2. (a) $0.1766$ (b) $1 - 0.5618 = 0.4382$ (c) $0.7583$

3. (a) $0.3585$ (b) $0.1887$ (c) $0.9841$

4. $\frac{48}{81} = 0.593$

5. (a) $0.2001$ (b) $0.5981$ (c) $0.2018$

**Summary**

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In general, if $X \sim \text{B}(n, p)$ then $P(X = x) = {}^{n}C_{x} \times p^{x} \times (1 - p)^{n-x}$

where $n$ is the number of trials
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