Coefficient of Restitution

Starter

1. **(Review of last lesson)** The masses of two particles, P and Q, are respectively 0.18 kg and 0.1 kg. They are moving directly towards each other at speeds of 4 m/s and 12 m/s respectively. After they collide the direction of motion of each particle is reversed and the speed of Q is 6 m/s. Find P's speed after impact.

Notes

Consider object A catching up and colliding with object B. Prior to the collision they had velocities u_A and u_B respectively and after the collision their velocities are v_A and v_B.
In order for A to catch up with B, $u_A > u_B$ and $u_A - u_B$ is called the approach speed
Assuming the objects do not coalesce, $v_A > v_B$ and $v_A - v_B$ is called the separation speed

Newton's law of impact (NLI)

Velocity of separation = constant × velocity of approach
The constant is called the ***coefficient of restitution***, e, where $0 \leq e \leq 1$.

- $e = 1 \Rightarrow$ perfectly elastic (no kinetic energy lost) (objects separate at same relative speed as they approached)
- $e = 0 \Rightarrow$ perfectly inelastic ($v_B = v_A$ i.e. the objects coalesce)
- Most problems will have $0 < e < 1$

N.B. Make sure you state the positive direction on your diagram.
Newton's law of impact is sometimes called Newton's experimental law.

NLI with two objects

e × relative speed of approach = relative speed of separation

$$e(u_A - u_B) = v_B - v_A \quad \text{or} \quad v_B - v_A = -e(u_B - u_A)$$

Unless $u_A > u_B$, the objects will not collide.
Unless $v_B > v_A$, the objects will not separate.

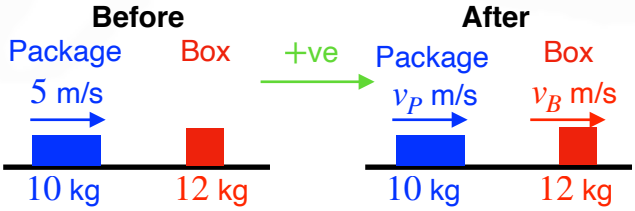
Solving problems involving the coefficient of restitution

1. Form one equation using the conservation of momentum, CoM.
2. Form a second equation using Newton's law of impact, NLI.

The introduction of NLI allows questions to have two unknowns.

E.g. 1 In a warehouse a package of mass 10 kg moving at 5 m/s collides with a stationary box of mass 12 kg. If the coefficient of restitution is 0.1, find the speed of each after the collision.

Working:



CoM: $10 \times 5 = 10v_P + 12v_B$ where $v_B > v_P$
$50 = 10v_P + 12v_B$
NLI: $0.1 \times 5 = v_B - v_P$
$0.5 = v_B - v_P$
Solve simultaneously: $v_B = 2.5$ $v_P = 2$
The speed of the box is 2.5 m/s and the speed of the package is 2 m/s.

E.g. 2 A metal ball of mass 70 grams is moving at 4 m/s when it collides with a wooden ball of mass 30 grams, which is moving in the same line of direction at 6 m/s. Given that $e = 0.5$, find the velocities of the balls after the collision.

One object hitting a fixed surface (e.g. a wall) at a right angle

Let u be the velocity of object before it hits the surface and v be the velocity with which it rebounds.

Using NLI: $e \times$ velocity of approach $=$ velocity of separation

$eu = -v$ since u and v are in opposite directions

i.e. $v = -eu$

Alternatively, using speeds: $v = eu$

E.g. 3 A ball hits a wall with speed 1.8 m/s and rebounds at 1.2 m/s. Find e.

Working: NLI: $1.8e = 1.2 \quad \Rightarrow \quad e = \frac{2}{3}$

E.g. 4 An ice hockey player hits a puck at 10 m/s which hits a wall 12 m away. Given that the coefficient of restitution is 0.6 and ignoring friction, find the time it takes for the puck to return to the player.

Video: Newton's law of restitution

Video: Collisions involving e

Video: Wall collisions involving e

Momentum involving e EQ

Solutions to Starter and E.g.s

Exercise

p72 3C Qu 1-8 (red 9—18)

Summary

Newton's law of impact: velocity of separation $= e \times$ velocity of approach where $0 \leq e \leq 1$.

- $e = 1 \Rightarrow$ perfectly elastic (no kinetic energy lost)
- $e = 0 \Rightarrow$ perfectly inelastic ($v_B = v_A$ i.e. the objects coalesce)

Make sure you state the positive direction on your diagram.

NLI with two objects: $e(u_A - u_B) = v_B - v_A$ or $v_B - v_A = -e(u_B - u_A)$

NLI with one object hitting a wall: $v = -eu$ where u and v are velocities

Solving problems involving the coefficient of restitution

1. Form one equation using the conservation of momentum, CoM.
2. Form a second equation using Newton's law of impact, NLI.