

## Combinations

### Starter

1. **(Review of previous material)**  
How many permutations of the letters of the word GROUP start with a vowel?
2. **(Review of last lesson)**  
Eight athletes run a 100m race. In how many ways can the first 3 places be filled?
3. How many different ways are there to **choose** the letters A, B and C given that we can choose one, two or three letters?  
**N.B.** The order you choose them in is not important.

### Notes

A **combination** is an **unordered** (i.e. how they are arranged is irrelevant) group chosen from the items in a set. Normally, you will be given a set of  $n$  items and you will be asked how many ways there are to choose  $r$  of these items. Obviously,  $r \leq n$ .

**E.g. 1** A museum has places to hang 3 paintings from the five they have available. How many different **combinations** are there i.e. position is not important?

**Working:** Let the paintings be A, B, C, D and E.

If the question was a permutations problem, we would have  ${}^5P_3 = \frac{5!}{2!}$  arrangements.

However, the actual arrangement of the strikers is unimportant (i.e. ABC = BAC).

So we have  $3!$  times too many and need to divide by this number.

The number of combinations is  $\frac{5!}{2!3!} = 10$

In general,

$$\text{Number of choices} = \frac{\text{Total number of permutations}}{\text{Number of permutations of each set}}$$

If a set of  $r$  objects is chosen from  $n$  unique objects there are  $\frac{n!}{(n-r)!r!}$  combinations (choices).

### Notation

This is given special notation,

$${}^nC_r = \frac{n!}{(n-r)!r!} = \frac{\text{Number available!}}{\text{Number not chosen! Number chosen!}}$$

**N.B.** Some books use different notation:  ${}^nC_r = {}_nC_r = \binom{n}{r}$

The latter is particularly silly as it looks like a column vector, although context should make it clear.

Use the  $nCr$  button on your calculator (on the Classwiz: SHIFT >> ÷)

**Special cases**

- ${}^n C_r = \frac{n!}{(n-r)!r!} = \frac{n!}{r!(n-r)!} = {}^n C_{n-r}$
- What does  $0!$  equal? Consider the question: "How many ways are there of choosing  $n$  objects from  $n$  available objects?"

A similar reasoning can be used for choosing no objects from  $n$  available objects.

- What does  ${}^n C_1$  mean in combination terms?
- What does  ${}^n C_{n-1}$  mean in combination terms?

**E.g. 2** The manager of a football team has a squad of 16 players. He needs to choose 11 to play in a match. How many teams can be chosen?

**E.g. 3** In how many ways can 5 students be chosen from a class of 20 if the form captain must be chosen?

**E.g. 4** In how many ways can a party of 10 children be divided into 2 groups of:  
(a) 6 and 4 children  
(b) 5 children each

**E.g. 5** Five cards are dealt without replacement from a standard pack of 52 cards. Find the probability that exactly 3 of the 5 cards are hearts.

**Difference between permutations and combinations**

Permutations  $\equiv$  arrangement of **all or some** of the items

All items:  $n!$

Some items:  ${}^n P_r = \frac{n!}{(n-r)!}$

Combinations  $\equiv$  **choosing a specific number of items** in any order,  ${}^n C_r = \frac{n!}{(n-r)!r!}$

**E.g. 6** Each question asks either for a number of combinations or for a number of permutations. Without attempting to find that number, decide whether you are looking for combinations or permutations.

- How many arrangements of the letters X, Y and Z are there?
- A team of 4 is to be chosen from nine players. How many different teams can be selected?
- If 8 records can be taken to a desert island, from a collection of one hundred records, how many different sets can be chosen?
- Five hundred raffle tickets are sold. When the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> prizes are drawn, in how many ways can the prizes be won?
- A museum lists its exhibits by code numbers of seven digits. How many code numbers are available?

- (f) A door is to be painted in two shades of green paint. If six suitable shades are available, in how many ways can the two shades be selected?
- (g) In how many ways can fifteen books be placed on a shelf?
- (h) A team has five strikers in the squad. The manager will choose two strikers to start the game and position is important.

[Video: Combinations](#)  
[Video: Combining combinations](#)

[Solutions to Starter and E.g.s](#)

### Exercise

p8 1C Qu 1i, 2i, 3-11, (12-14 red)

### Summary

A **combination** is an unordered (i.e. how they are arranged is irrelevant) group chosen from the items in a set.

$${}^n C_r = \frac{n!}{(n-r)!r!} = \frac{\text{Number available!}}{\text{Number not chosen! Number chosen!}}$$