Complex Solutions to Polynomial Equations

Starter

1. **(Review of last lesson)**
   Express the $z^3 + 3z^2 - 8z - 30$ polynomials as the products of a linear and a quadratic factor with real coefficients. Hence solve the equation when it is equal to zero.

2. **In this question you must show detailed reasoning.** (i.e. do not use your calculator)
   By solving the quadratic $z^2 - 2z + 5 = 0$ using the formula, express it as the product of two linear factors.

Notes

From the starter and from the quadratic formula, it is clear that the ***complex roots*** of a quadratic with real coefficients come in ***conjugate pairs***.

Therefore, **if $a + b\text{i}$ is root then so is $a - b\text{i}$** and vice versa.

Roots and factors

If a is a root of a polynomial equation then $x - a$ is a factor.
In general:

$x = \frac{b}{a}$ is a root $\Leftrightarrow$ $ax - b$ is a factor

$z = a \pm b\text{i}$ are roots $\Leftrightarrow$ $x - (a + b\text{i})$ and $x - (a - b\text{i})$ are factors
$\Leftrightarrow$ $x - a - b\text{i}$ and $x - a + b\text{i}$ are factors

E.g. 1 Find the equation of the quadratic which has one root $3 - 5\text{i}$ and whose coefficient of the squared term is 1.

Working: The quadratic equation is of the form $z^2 + bz + c = 0$
If $3 - 5\text{i}$ is a root, then $3 + 5\text{i}$ is also a root.

$$(z - (3 - 5\text{i}))(z - (3 + 5\text{i})) = z^2 - 6z + 3^2 + 5^2$$
$$= z^2 - 6z + 34$$

So the equation is $z^2 - 6z + 34 = 0$

N.B. Remember $(a + b\text{i})(a - b\text{i}) = a^2 + b^2$

E.g. 2 The equation $z^3 - 7z^2 + 16z - 10 = 0$ has a root $3 + \text{i}$. Find the other two roots of the equation.

Questions can also require the use of the factor theorem, similar to question 1 from the starter.

E.g. 3 **In this question you must show detailed reasoning.**
Solve the equation $z^3 - 3z^2 + z + 5 = 0$.

Video: Solving cubic equations
Video: Solving quartic equations

Solutions to Starter and E.g.s

Exercise
p147 5B Qu 1i, 2i, 3i, 4-9

Summary
Complex roots appear as conjugate pairs: if $a + b\text{i}$ is root then so is $a - b\text{i}$

$x = \dfrac{b}{a}$ is a root $\Leftrightarrow$ $ax - b$ is a factor

$z = a \pm b\text{i}$ are roots $\Leftrightarrow$ $x - (a + b\text{i})$ and $x - (a - b\text{i})$ are factors