Components of acceleration (a general model) (A2)

Starter

1. **(Review of last lesson)** A particle is threaded on a smooth circular wire of radius, r. Starting from a point horizontally level with the centre, the particle needs an initial velocity of v to reach the highest point. Calculate the percentage increase in velocity needed if the particle starts from the lowest point of the circle.

Notes

When the velocity changes as a particle moves around the circle there is ***tangential acceleration*** as well as ***radial acceleration***.

Tangential acceleration, a_t — acceleration ***along the direction of motion:*** $a_t = \frac{dv}{dt}$

a_t can be considered the rate of change of velocity.

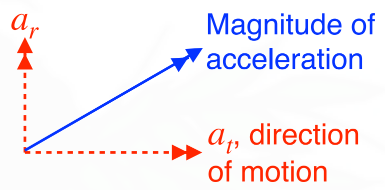
Since $v = r\omega$: $a_t = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\dot{\omega}$ or $a_t = r\ddot{\theta}$ since $\omega = \frac{d\theta}{dt}$

In general: $a_t = \frac{dv}{dt} = r\dot{\omega} = r\ddot{\theta}$

a_t is similar to normal acceleration and can be considered as the rate of change of velocity.

Radial acceleration, a_r — acceleration ***towards the centre:*** $a_r = \frac{v^2}{r} = v\omega = r\omega^2$

The ***magnitude of the acceleration*** $= \sqrt{a_r^2 + a_t^2}$



N.B. The v in the formulae for radial and tangential acceleration is the same v.

Solving problems

There are several strategies that can be used:

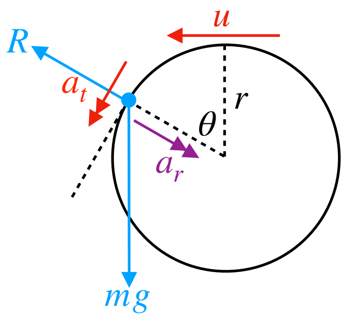
- Use the conservation of energy in order to find v^2 and hence a_r.
- Use $F = ma$ radially i.e. $F = ma_r$.
- Use $F = ma$ tangentially i.e. $F = ma_t$.
- A value or expression for a_t can be integrated to find v.

E.g. 1 Consider a bead of mass m threaded on a smooth of wire in the shape of a circle of radius r. Initially the bead is at the highest point and has speed u. After the bead has rotated through an angle θ find, in terms of g, r and θ:

(a) the radial acceleration

(b) the tangential acceleration

Working: (a) $a_r = \dfrac{v^2}{r}$



New KE = Initial KE + Loss in GPE

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg(r - r\cos\theta)$$

$$v^2 = u^2 + 2gr(1 - \cos\theta)$$

$$a_r = \frac{u^2 + 2gr(1 - \cos\theta)}{r}$$

The radial acceleration is $\dfrac{u^2 + 2gr(1 - \cos\theta)}{r}$

N.B. Using $F = ma$ radially would not work since we do not know R.

(b) Using $F = ma$ tangentially: $mg\sin\theta = ma_t$

$$a_t = g\sin\theta$$

The tangential acceleration is $g\sin\theta$

E.g. 2 A cyclist decreases her speed uniformly from 36 km/h to 27 km/h in 3 seconds while on the circular part of a horizontal track of radius 20 m. Find:

(a) the tangential acceleration

(b) an expression for the radial acceleration in terms of t

(c) the magnitude of the acceleration when $t = 1$.

E.g. 3 A motorcyclist is rounding a bend of radius 20 metres. She enters the bend travelling at 10 m/s, and increase speed at a constant rate of 2 m/s² for each of the next 3 seconds. Find the magnitude of the acceleration and the angle it makes with the direction of motion at:

(a) the start of this time i.e. $t = 0$

(b) the end of this time i.e. $t = 3$

Solutions to Starter and E.g.s

Exercise

p240 9B Qu 1-5 (red 6)

Summary

Tangential acceleration — acceleration along the direction of motion $a_t = \frac{dv}{dt} = r\dot{\omega} = r\ddot{\theta}$

Radial acceleration, a_r — acceleration towards the centre: $a_r = \frac{v^2}{r} = v\omega = r\omega^2$

Magnitude of the acceleration $= \sqrt{a_r^2 + a_t^2}$

Solving problems:

- Use the conservation of energy in order to find v^2 and hence a_r.
- A value or expression for a_t can be integrated to find v.
- Use $F = ma$ radially i.e. $F = ma_r$.
- Use $F = ma$ tangentially i.e. $F = ma_t$.