Conical pendulums

Starter

1. **(Review of last lesson)** A car, of mass 800 kg, is travelling at a steady speed of 15 m/s round a roundabout of radius 20 m on a flat road. Calculate:
   (a) the magnitude of the acceleration
   (b) the sideways force on each wheel assuming it is the same for each wheel.

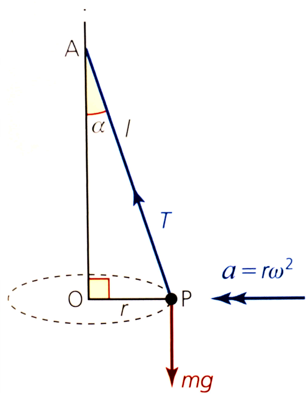
2. **(Review of last lesson)** (A2 coefficient of friction). A small block A, of mass m kg, lies on a horizontal disc which is rotating about its centre B at 3 rad/s and $AB = 0.8$ m. If the block does not move relative to the disc, find the least value of coefficient of friction between the block and the disc.

Notes

Situations where the forces are not acting horizontally will now be considered. The first example is conical pendulums i.e. where the string or rod sweeps out a cone as it rotates.

Conical pendulum

E.g. 1 A particle P, of mass m kg, is attached to a light inextensible string of length l m. The particle moves in a horizontal circle of radius r m. The string is fixed at A and makes an angle of α with the downward vertical. The tension in the string is T N.
The two forces acting on the particle are the weight and the tension in the string. By resolving vertically and using $F = ma$ horizontally, derive a formula for the angular speed, ω, in terms of g, l and α.



Working:

$R(\uparrow)$: $T\cos\alpha = mg$

$F = ma(\rightarrow)$: $T\sin\alpha = ma_r$

Since $a_r = r\omega^2$: $T\sin\alpha = mr\omega^2$

Using trigonometry: $r = l\sin\alpha$

$\therefore T\sin\alpha = m \times l\sin\alpha \times \omega^2$

$T = ml\omega^2$

Substituting into the first equation: $ml\omega^2\cos\alpha = mg$

$$\omega = \sqrt{\frac{g}{l\cos\alpha}}$$

From the formula:

- The ***mass*** of the particle has ***no effect on angular speed***
- The ***angular speed increases when*** the ***string is shortened***
- If $\alpha \geq 90°$, $\cos\alpha \leq 0$ so $\dfrac{g}{l\omega^2} \leq 0$ which is impossible so $0 < \alpha < 90°$.

Solving problems involving conical pendulums

- Use $F = ma$ radially i.e. $F = ma_r$.
- Use $a_r = r\omega^2$ or $a_r = v\omega$ or $a_r = \dfrac{v^2}{r}$ depending on the situation.
- Resolve vertically, $R(\uparrow)$.

E.g. 2 A particle is attached to one end of a light inextensible string of length 1.6 m. The other end of the string is attached to a fixed point. The particle moves, at constant speed, in a horizontal circle, with the string inclined at 55° to the vertical. Calculate the tangential speed of the particle.

Working: Use $F = ma$ radially: $T\sin 55 = ma_r$

$R(\uparrow)$: $T\cos 55 = mg$

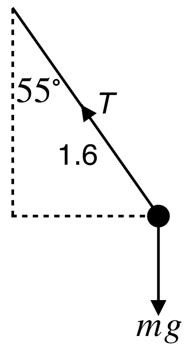
Dividing: $\dfrac{T\sin 55}{T\cos 55} = \dfrac{ma_r}{mg}$

$\therefore a_r = g\tan 55$ since $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Replace a_r by $\dfrac{v^2}{r}$: $\dfrac{v^2}{1.6} = g\tan 55$

$v = \sqrt{1.6g\tan 55} \approx 4.28$ m/s

The tangential speed of the particle is 4.28 m/s (3 s.f.)



E.g. 3 In a simple model of a 'rotating swing', a particle of mass 30 kg is attached to one end of a light inextensible rope of length 2 m. The other end is attached to a fixed point O. The particle moves in a horizontal circle at a constant angular speed of 3 rad/s. The rope is inclined at a constant angle θ to the vertical. Find θ.

Video: Conical pendulum
Video: Motion on inside of cones
Video: Motion inside hemisphere

Solutions to Starter and E.g.s

Exercise
p96 4C Qu 1i, 2i, 3, 5, 7, 8

Summary
Solving problems involving conical problems:

- Use $F = ma$ radially i.e. $F = ma_r$.
- Use $a_r = r\omega^2$ or $a_r = v\omega$ or $a_r = \dfrac{v^2}{r}$ depending on the situation.
- Resolve vertically, $R(\uparrow)$.