Cubic and Quartic equations

Starter

1. **(Review of last lesson)** Write down a quadratic equation with integer coefficients such that the sum of the roots is $\frac{1}{3}$ and the product is $\frac{1}{2}$.

2. Let the roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$ be alpha (α), beta (β) and gamma (γ). Therefore, $ax^3 + bx^2 + cx + d \equiv a(x - \alpha)(x - \beta)(x - \gamma)$.
By expanding and equating coefficients, express the following in terms of a, b, c and d.
(a) $\alpha + \beta + \gamma$ (b) $\alpha\beta + \beta\gamma + \gamma\alpha$ (c) $\alpha\beta\gamma$

Notes

Cubic equations

For the cubic equation $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a} \qquad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \qquad \alpha\beta\gamma = -\frac{d}{a}$$

E.g. 1 Find the simplest cubic with roots 2, 3, and 4.

E.g. 2 Expand $(\alpha + \beta + \gamma)^2$.

E.g. 3 The cubic equation $2x^3 + 3x^2 + 4x + 5 = 0$ has roots α, β and γ. Find the values of:
(a) $\alpha^2 + \beta^2 + \gamma^2$ (b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

E.g. 4 Write down the equation whose roots are α, β and γ where $\alpha + \beta + \gamma = 6$, $\alpha\beta + \alpha\gamma + \beta\gamma = 11$ and $\alpha\beta\gamma = 6$. Hence solve the equation.

Quartic equations

It can be shown for the quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ that:

$$\alpha + \beta + \gamma + \delta = -\frac{b}{a} \qquad \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$$

$$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a} \qquad \alpha\beta\gamma\delta = \frac{e}{a}$$

Video: Sum and product of roots (cubic)
Video: Sum and product of roots (quartic)

Solutions to Starter and E.g.s

Exercise

p153 5D Qu 1, 2ac, 3-9

Summary

For the cubic equation $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a} \qquad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \qquad \alpha\beta\gamma = -\frac{d}{a}$$

For the quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$:

$$\alpha + \beta + \gamma + \delta = -\frac{b}{a} \qquad \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$$

$$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a} \qquad \alpha\beta\gamma\delta = \frac{e}{a}$$