Determinants of 2 by 2 Matrices

Starter

1. Let $\mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 & 2 & 3 \\ -2 & 3 & -3 \end{pmatrix}$ and $\mathbf{C} = (2 \quad 3 \quad 1)$.

Calculate the following products if possible:

(a) $\mathbf{AB}$ (b) $\mathbf{BA}$ (d) $\mathbf{CA}$ (e) $\mathbf{BC}$

Notes

The ***determinant*** of a matrix is a number associated with a matrix and is analogous to the magnitude of a vector.

The determinant of the 2 by 2 matrix $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is given by $ad - bc$.

$\det \mathbf{M} = ad - bc$ or $|\mathbf{M}| = ad - bc$

N.B. Only square matrices have determinants.

Notation

The capital Greek letter delta, Δ, is often used as shorthand for the determinant.
A matrix with straight brackets indicates the determinant

i.e. $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ ***...or...*** $\Delta = \left| \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right| = ad - bc$

Singular matrices

Matrices whose ***determinant equals zero*** are called ***singular matrices***.

E.g. 1 Calculate the determinants for: (a) $\mathbf{A} = \begin{pmatrix} 3 & -5 \\ 4 & 7 \end{pmatrix}$ (b) $\mathbf{B} = \begin{pmatrix} -6 & 2 \\ 4 & 9 \end{pmatrix}$

Working: (a) $\det \mathbf{A} = \left| \begin{pmatrix} 3 & -5 \\ 4 & 7 \end{pmatrix} \right| = 3 \times 7 - 4 \times (-5) = 41$

Video: Determinant of a 2 by 2 matrix

Solutions to Starter and E.g.s

Exercise

p16 1C Qu 1i, 2, 3

Summary

The ***determinant*** is a useful number associated with a matrix.

$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

$\det \mathbf{AB} = \det \mathbf{A} \times \det \mathbf{B}$