Factorising Polynomials

Starter

1. **(Review of last lesson)**
   Let $z_1 = \sqrt{3} - \mathrm{i}$ and $z_2 = -\sqrt{2} + \sqrt{2}\mathrm{i}$. By giving your answers $0 \leq \theta < 2\pi$, find:
   (a) Arg $(z_2 \times z_1)$
   (b) Arg $(z_2 \div z_1)$
   (c) Arg (z_1^2)

2. **(Review of GCSE material)** Factorise the quadratics:
   (a) $x^2 - 4x - 12 = 0$ (b) $2x^2 + 7x - 15 = 0$

3. **(Review of AS material)** The polynomial $p(x) = x^3 + ax^2 + 2x + b$ is divisible by $x + 1$ and has a factor $x - 2$. Find the values of a and b.

Notes

Roots and factors

If a is a root of a polynomial equation then $x - a$ is a factor.
In general:

$$x = \frac{b}{a} \text{ is a root} \quad \Leftrightarrow \quad ax - b \text{ is a factor}$$

Factor theorem (a reminder)

For the polynomial $f(x)$, if $f(a) = 0$ then $x = a$ is a root of the equation.
It also follows that $x - a$ is a factor.

Similarly: $$f\left(\frac{b}{a}\right) = 0 \quad \Leftrightarrow \quad ax - b \text{ is a factor}$$

E.g. 1 Express $z^3 + z - 10$ as the product of a linear and a quadratic factor with real coefficients and hence solve the equation $z^3 + z - 10 = 0$.

Extending the difference of two squares

E.g. 2 (a) Given that $a^2 - b^2 = (a + b)(a - b)$, express $a^2 + b^2$ as the product of two complex numbers
(b) Hence factorise $z^4 - 16$ into four linear factors.

Video: Solving quadratic equations with complex roots

Solutions to Starter and E.g.s

Exercise

p143 5A Qu 1i, 2i, 3i, 4-8

Summary

$$x = \frac{b}{a} \text{ is a root} \quad \Leftrightarrow \quad ax - b \text{ is a factor}$$

$$f\left(\frac{b}{a}\right) = 0 \quad \Leftrightarrow \quad ax - b \text{ is a factor}$$