

## Finding an Equation with Given Roots

### Starter

1. (Review of last lesson)

Let the roots of the equation  $x^3 - 6x + 2 = 0$  be  $\alpha$ ,  $\beta$  and  $\gamma$ . Find the value of:

- (a)  $4\alpha + 4\beta + 4\gamma$  (b)  $\alpha^2 + \beta^2 + \gamma^2$   
(c)  $(\beta - \gamma)^2 + (\gamma - \alpha)^2 + (\alpha - \beta)^2$  (d)  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

2. The quadratic equation  $x^2 + 5x + 7 = 0$  has roots  $\alpha$  and  $\beta$ . Without calculating  $\alpha$  and  $\beta$ , find an equation with roots  $2\alpha$  and  $2\beta$ .

### Notes

There are two main methods available for question 2 of the starter. Next lesson will look at the substitution method.

#### Roots method

$$x^2 + 5x + 7 = 0 \Rightarrow \alpha + \beta = -5 \text{ and } \alpha\beta = 7$$

Sum of roots of new equation:  $2\alpha + 2\beta = 2(\alpha + \beta) = 2 \times -5 = -10$

Product of roots of new equation:  $2\alpha 2\beta = 4\alpha\beta = 4 \times 7 = 28$

So new equation is  $x^2 + 10x + 28 = 0$

**E.g. 1** The equation  $x^2 + 2x + 5 = 0$  has roots  $\alpha$  and  $\beta$ . Use the “roots method” to find equations with integer coefficients which have the following roots.

- (a)  $3\alpha$  and  $3\beta$  (b)  $\alpha + 1$  and  $\beta + 1$  (c)  $\frac{1}{\alpha}$  and  $\frac{1}{\beta}$

The method also works for cubics and quadratics.

**E.g. 2** The equation  $2x^3 - 5x^2 + 3x + 4 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ . Use the “roots method” to find equations with integer coefficients which have the following roots.

- (a)  $3\alpha$ ,  $3\beta$  and  $3\gamma$  (b)  $\alpha - 1$ ,  $\beta - 1$  and  $\gamma - 1$

Other questions can be asked that require you to deduce the relationship between coefficients.

**E.g. 3** One root of the equation  $ax^2 + bx + c = 0$  is the reciprocal of the other. Prove that  $c = a$ .

**E.g. 4** Find the quadratic equation  $ax^2 + bx + c = 0$ , one root is twice the other. Prove that  $2b^2 = 9ac$ .

**Video:** [Finding equation given roots](#)  
**Video:** [Finding equation given roots example](#)

[Solutions to Starter and E.g.s](#)

### Exercise

p159 5E Qu 1i, 2i, 3-9, (10-12 red)