Finding dimensions from units and derivatives and predicting formulae

Starter

1. A formula is given by $t = kl^{\alpha}m^{\beta}g^{\gamma}$ where k is dimensionless, t is the time for 1 period, l is the length of string, m is the mass at the end of the string and g is the acceleration due to gravity. Use dimension analysis to determine the values of the powers α, β and γ.

Notes

Dimensional analysis can be used to predict formulae by equating dimensions of all the terms in the formula.

E.g. 1 For any sphere the terminal velocity, v, is thought to depend on its radius, r, its weight, mg, and the viscosity of the liquid η (in kg m^{-1} s^{-1}).

(a) Write down a formula for v as the product of unknown powers of r, mg, η and a dimensionless constant k.

(b) Find the powers of r, mg and η.

Dimensions of derivatives

Let r be the radius of a circle

First derivatives: $\left[\frac{dr}{dt}\right] = \left[\frac{r}{t}\right] = \mathrm{LT}^{-1}$

Second derivatives: $\left[\frac{d^2r}{dt^2}\right] = \left[\frac{d}{dt}\left(\frac{dr}{dt}\right)\right] = \left[\frac{r}{t^2}\right] = \mathrm{LT}^{-2}$

In each case the dimensions of the numerator are the same as that of the variable.
For second derivatives, note that the denominator is raised to the power of 2 but the numerator is not.

E.g. 2 The rate of flow of the volume R of a liquid with viscosity η (in kg m^{-1} s^{-1}) through a cylindrical pipe of length l and an internal radius r is believed to be of the form $R = k\eta^{w}l^{x}r^{y}p^{z}$ where k is a dimensionless constant and p (in Nm^{-2}) is the pressure difference between the ends of the pipe. Using dimension considerations, find:

(a) the values of w and z

(b) the relationship between x and y.

Video: Dimensional analysis

Video: Modelling using dimensional analysis

Solutions to Starter and E.g.s

Exercise

p41 2C Qu 1-10
p42 2D Qu 1 (table of dimensions)

Summary

Dimensional analysis can be used to predict formulae by equating dimensions of all the terms in the formula.

Dimensions of derivatives:
The dimensions of the numerator are the same as that of the variable.
For second derivatives, the denominator is raised to the power of 2 but the numerator is not.