Impulsive tension in strings

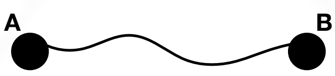
Starter

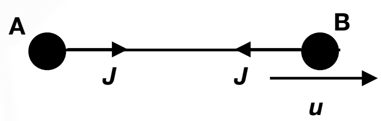
1. (Review of last lesson) A truck of mass 300 kg travelling at 4 m/s collides with a second truck travelling at 1 m/s in the opposite direction. The first truck is brought to rest by the collision. If $e = 0.4$, find the mass of the second truck.

Notes

Impulsive tensions acting in line with the string

Consider two particles A and B connected by a light inextensible string of length l and lying on a smooth table. The distance between A and B is less than l so that the string is not taut.





If B is projected away from A it will move with a constant speed until $AB = l$. At that instant the string jerks tight and suddenly exerts ***equal and opposite impulsive tensions***, J, on A and B.

The impulse on A pulls it forward, while the impulse on B acts in the opposite direction and slows it down. Since the string is taut, the particles will initially ***move at the same speed***, v.

These impulsive tensions cause A and B to experience equal and opposite changes in momentum. Therefore, just as in the case of a collision, the total momentum of the system is unchanged i.e. the principle of conservation of linear momentum can be applied.

Conservation of linear momentum: $m_B u = (m_A + m_B)v$

Impulse: Impulse on $A = m_A v - 0 = m_A v$

Impulse on $B = m_B v - m_B u$

N.B. These impulses are equal but act in opposite direction. Therefore, before equating we would need to change the sign of one of them. Then we would get the equation of the conservation of momentum.

When solving problems

- Use the conservation of momentum.
- Remember both particles will have the same velocity when the string becomes taut.
- To find impulsive tension consider ***either*** of the particles ***individually.***

E.g. 1 Two particles A and B, joined by light inextensible string, are lying together on a smooth horizontal plane. The masses of A and B are 1 kg and 1.5 kg respectively. A is projected away from B with a speed of 5 m/s. Find

(a) the speed of each particle after the string jerks taut,

(b) the impulse in the string when it jerks taut.

Working: (a) Momentum before $= 1 \times 5 = 5$

Momentum after $= (1 + 1.5)v$

$(1 + 1.5)v = 5$

$v = 2$ m/s

The speed of each particle after the string jerks taut is 2 m/s.

(b) ***Either*** Impulse $=$ change in momentum for B

$=$ final momentum $-$ initial momentum

$= 1.5 \times 2 - 0 = 3$ Ns

or Impulse $=$ change in momentum for A

$=$ final momentum $-$ initial momentum

$= 1 \times 2 - 1 \times 5 = -3$ Ns

N.B. The negative sign for A shows the impulse is acting in the opposite direction.

E.g. 2 Two particles A and B of equal mass m are connected by a light inextensible string of length l. Initially they are held at rest in the air, side by side. A is then released from rest.

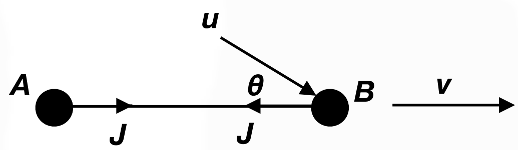
(a) Find, in terms of l and g, A's speed just as the string is about to jerk taut.

(b) If B is released at this instant find, in terms of l and g, the common speed with which A and B together begins to move.

(c) Find the impulse in the string.

Impulsive tensions acting at an angle to the string

What happens if the original direction of the moving particle is not in line with the string?



After the string becomes taut, the velocity of A *and* component of velocity of B ***parallel to the string*** are v.

The impulse will still act along the string. Therefore, the velocity of B perpendicular to the string is unchanged i.e. $u \sin \theta$.

We then use the conservation of momentum parallel to the string in order to find the component of velocity in that direction.

Conservation of momentum parallel to string: $m_B u \cos \theta = (m_A + m_B)v$

Impulse (best to consider only A): A: Impulse on $A = m_A v - 0 = m_A v$

E.g. 3 Two particles, A and B, of masses 1.5 kg and 2.5 kg respectively are attached by a light, slack in extensible string and lie on a smooth horizontal surface. Particle A is initially at rest while particle B is projected at a speed of $4\sqrt{2}$ m/s and when the string becomes taut it makes an angle of $45°$ with the string. Find:

(a) the speed of A just after the string is taut
(b) the speed and direction of B just after the string is taut
(c) the impulse in the string

Working: (a) CoM parallel to string: $2.5 \times 4\sqrt{2}\cos 45 = (1.5 + 2.5)v$

Velocity of A is $v = 2.5$ m/s

The speed of A just after the string is taut is 2.5 m/s.

(b) For B: Velocity parallel to string is 2.5 m/s

Velocity perpendicular to string is the same as before the string became taut i.e. $4\sqrt{2}\sin 45 = 4$

So velocity of B is $\sqrt{4^2 + 2.5^2} = 4.717$ m/s

Direction is $\tan^{-1}\dfrac{4}{2.5} = 58.0°$ to line of the string

(c) Impulsive tension for A = $1.5 \times 2.5 - 0 = 3.75$ Ns

E.g. 4 A light inextensible string has particles A and B, each of mass 400 g, attached to either end. The string and particles rest on a smooth table. The particle at A is hit with an impulse of 8 Ns at an angle $30°$ to the string, which is straight but not taut and at rest. What is the velocity of B after the impulse at the instant the string becomes taut?

N.B. Each question asks for the velocity of the particles immediately after the string becomes taut. This is because in most real-life cases the string will become slack again once the particles start moving.

Video: Impulsive tension example

Solutions to Starter and E.g.s

Exercise
p219 8C Qu 7-9 (red)

Summary
When solving problems with impulsive tensions:

- Use the conservation of momentum
- Remember both particles will have the same velocity when the string becomes taut
- To find impulsive tension consider either of the particles individually

Impulsive tensions acting at an angle to the string:
The impulse will still act along the string. Use the conservation of momentum parallel to the string in order to find the component of velocity in that direction.
Conservation of linear momentum parallel to string: $m_B u \cos\theta = (m_A + m_B)v$
Impulse (best to consider only A): A: Impulse on A = $m_A v - 0 = m_A v$