Induction and inequalities

Starter

1. Prove that $4^n + 5^n + 6^n$ is divisible by 15 by mathematical induction, when n is an odd positive integer.

Notes

The final proof by induction involves inequalities.
$A > B$ can be proved by proving $A - B > 0$ (see 2nd example below)

E.g. 1 Prove that $2^n > n$ for all natural numbers n.

Working:

(Proposition)
Let $P(n)$ be the proposition that $2^n > n$ for all natural numbers n.
(Prove the basic case)
When $n = 1$: $2^1 = 2 > 1$ which is true
Therefore $P(1)$ is true.
(Inductive step)
Assume that $P(k)$ is true i.e. $2^k > k$
(Inductive step — consider the next term)
Need to prove $P(k+1)$ is true i.e. $2^{k+1} > k+1$
(Inductive step — algebraic manipulation)
$P(k+1)$: $2^{k+1} = 2 \times 2k$
$> 2 \times k$ assuming $P(k)$ is true
$= k + k$
$\geq k + 1$ since $k \geq 1$
(Completion)
But this is $P(k)$ with k replaced by $k+1$.
Therefore, if $P(k)$ is true, then $P(k+1)$ is also true.
$P(1)$ is true and if $P(k)$ is true, then $P(k+1)$ is true. Using the principle of mathematical induction, $P(n)$ is true for all positive integers.

E.g. 2 Prove that $4^{n-1} > n^2$ for $n \geq 3$.

N.B. $(n+1)! = (n+1) \times n!$

E.g. 3 Prove that $n! > 2^n$ for $n \geq 4$.

Video A: Proof by induction (inequalities)
Video B: Proof by induction (inequalities)

Solutions to Starter and E.g.s

Exercise
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Summary
$A > B$ can be proved by proving $A - B > 0$
$(n+1)! = (n+1) \times n!$