Introduction to Hypothesis Testing

Starter

1. **(Review of last lesson)** Without using the special function on your calculator, find the PMCC for $\sum x_i = 357.7$, $\sum y_i = 278.7$, $\sum x_i^2 = 11696.95$, $\sum y_i^2 = 7119.71$, $\sum x_i y_i = 8396.1$ and $n = 12$. Comment on your findings.

2. **(Review of last lesson)**
Using the special function on your calculator, find the PMCC for the following data:

x	1020	1032	1028	1034	1023	1038
y	320	335	345	355	360	380

Comment on your findings.

Notes

Hypothesis testing will be covered in much great depth in the AS Maths course but without an understanding of what one is, the next sections will be too difficult.

It was invented by Ronald Fisher following a tea party when another guest, Dr Muriel Bristol, said she could tell whether tea was put in before the milk and vice verse. She guessed 8 out of 8 cups correctly.

The concept of a hypothesis test can be difficult to grasp so it important to spend time on the basics before doing any calculations.

A hypothesis test is analogous to a court case, where two fundamental principles exist:

- The defendant is ***innocent until proven guilty***.
- The proof must be ***beyond all reasonable doubt***.

The jury must answer the following question:

What is the probability of this evidence being true and the defendant being innocent?

If the probability is very small, the jury would ***reject*** the assumption of innocence.

If the probability is large, the jury ***would not reject*** the assumption of innocence.

That is why defence attorneys aim to cast ***reasonable doubt*** on the evidence.

An example will help illustrate the situation.

Stolen bike

Last month in Ripon a very expensive carbon bike was stolen and a shady character called Mr White has been arrested and is in court after being accused of the theft.

The prosecution present 3 pieces of evidence. Remember, the jury need to answer this question:

What is the probability of this evidence being true and the defendant being innocent?

Evidence 1: Mr White has often talked about wanted a very expensive bike (evidence provided by Mrs White)

Mr White's lawyer would argue that there is nothing unusual in a cycling enthusiast wanting to have a more expensive bicycle.

Question 1: What is the probability of this evidence being true and the defendant being innocent?

High

So this evidence is dismissed as unimportant, but Mr White's lawyer hands Mrs White his business card as divorce proceedings look imminent.

Evidence 2: An eye-witness spotted a man with a beard walking out of the garage where the bike was housed in the dead of night.

Mr White's lawyer would argue that beards are popular with highly fashionable young man and so casts reasonable doubt on this evidence.

Question 2: What is the probability of this evidence being true and the defendant being innocent?

Medium

Evidence 3: The stolen bike was found in Mr White's garage.

Mr White's lawyer spins a yarn about the real thief breaking into the garage, of which there is no evidence, and placing the bicycle there for safe keeping or to frame Mr White, possibly by an ex-student after a poor grade.

Question 3: What is the probability of this evidence being true and the defendant being innocent?

Very low and off to the stocks goes Mr White

Comparison with statistics
Trial ≡ Hypothesis test
Defendant is innocent ≡ **Null hypothesis,** H_0 — assumed to be true
Defendant is not innocent ≡ **Alternative hypothesis,** H_1
Level of doubt ≡ **Significance level** — usually 5% (though 1% and 2% are also common)
Evidence ≡ **Test statistic** provided by the data

The significance level sets the amount of reasonable doubt that we are willing to accept. So a hypothesis test done at the 1% level generally has more weight than one carried out at the 5% level.

Careful use of language
At the end of a hypothesis test, the conclusion is expressed as "**reject** the null hypothesis, H_0" and "**do not reject** the null hypothesis H_0". We do not say that something is true or false. This is possibly where the court room and hypothesis tests diverge.

"There is evidence not to reject H_0" or "There is no evidence to reject H_0" means ***based on the data set we currently have***, this is the conclusion we would make. However, another dataset may lead us to reach a different conclusion (equivalent to an appeals process in the courts).

Solutions to Starter and E.g.s

Exercise
No exercise

Summary

	Hypothesis testing	Court case
Null Hypothesis	H_0: p = "The sample could come from the population i.e. the status quo"	The defendant is innocent. "Innocent until proven guilty".
Alternative hypothesis	H_1: p > ***or*** H_1: p < ***or*** H_1: p ≠	The defendant is guilty.
Evidence	The value from the sample - the test statistic	Provided by the police - witnesses, fingerprints etc.
Significance level (% of a false positive	The amount of doubt you are willing to allow, usually 5%, 2% or 1%.	Beyond reasonable doubt
Key questions	Does the test statistic (value from the sample) lie in the critical region?	If the evidence is true, how likely is it that the defendant is innocent?

False positive — reject the null hypothesis when it should be accepted
False negative — accept the null hypothesis when it should be rejected