Keeping Objects Together or Separated

Starter

1. **(Review of previous material)** A team of 4 is chosen at random from 5 girls and 6 boys.
   (a) In how many ways can the team be chosen if
      (i) there are no restrictions;
      (ii) there must be more boys than girls
   (b) Find the probability that the team contains only 1 boy.

Notes

We are back to arranging items i.e. permutations but now there are restrictions on the arrangement: either certain items must be kept together or they must be kept apart.

Items kept together

If items must be kept together they can be considered as one item, when looking at the overall arrangement. We must also consider the arrangement of the items that must be kept together.

E.g. 1 Find the number of ways of arranging 6 women and 3 men to stand in a row so that all three men are standing together.

Working: Consider the 3 men as a single unit: they can be arranged in $3!$ ways
The 6 women and group of men constitute 7 units.
There are $7!$ ways of arranging 7 units.
Total ways $= 3! \times 7! = 30240$

E.g. 2 Nine different flower pots are to be arranged in a row. If the rose, daffodil, orchid and tulip pots must all be together, how many arrangements are possible?

Working: Consider the rose, daffodil, orchid and tulip (4 items) pots as one item.
There are now 6 items (5 pots plus 1 group) to arrange: $6!$
The rose, daffodil, orchid and tulip (4 items) pots can be arranged in $4!$ ways
Total arrangements $= 6! \times 4! = 17280$

E.g. 3 There are n objects that need to be arranged in a row. If r of them must all be together, where $r < n$, how many arrangement are possible?

Working: The r objects that must be together are considered 1 item
There are now $n - r + 1$ objects to arrange: $(n - r + 1)!$ ways
The r objects can be arranged in $r!$ ways
Total arrangements $= (n - r + 1)! \times r!$

Video: Permutations (items together)
Video: Permutations (items at ends)

Items kept separate

When objects must be separated there are two methods we can use. Either we can:
(a) consider how many gaps this creates
(b) subtract the ways of being together away from the total number of arrangements.

Let's consider the gap method.

E.g. 4 Find the number of ways of arranging 6 women and 3 men to stand in a row so that no two men are standing together.

Working: Consider the women standing in a row:

$W_1 \quad W_2 \quad W_3 \quad W_4 \quad W_5 \quad W_6$

There are 7 gaps where the men could stand so they are not together:

$G_1 \quad W_1 \quad G_2 \quad W_2 \quad G_3 \quad W_3 \quad G_4 \quad W_4 \quad G_5 \quad W_5 \quad G_6 \quad W_6 \quad G_7$

i.e. the 3 men have 7 gaps to choose from: ${}^7C_3$

The 3 men can be arranged in $3!$ ways

The 6 women can be arranged in $6!$ ways

Total ways $= {}^7C_3 \times 3! \times 6! = 151200$

E.g. 5 There are 3 boys and 4 girls in a team. The team photo has the players in a row and the boys must be separated.

(a) After removing the boys, how many gaps are there around and between the girls?

(b) How many ways can the 3 boys choose these gaps?

(c) How many ways can the 3 boys be arranged?

(d) How many ways can the 4 girls be arranged?

(e) What is the total number of arrangement of the team photo?

E.g. 6 The word UNCOPYRIGHTABLE is one of two words of 15 letters with no repeated letters. How many permutations are of the letters of UNCOPYRIGHTABLE if the vowels must be separated?

E.g. 7 A group of n distinct objects are to be arranged in a row but r of these objects must be separated. How many different arrangements are there?

Success Criteria - keeping objects separate

Method 1 — gap method

1. Write down the number of gaps.
2. Write down the number of ways the separated object can choose the gaps.
3. Write down the number of ways the separated objects can be arranged.
4. Write down the number of ways the non-separated objects can be arranged.
5. Multiply your three answers to step 2, 3 and 4 together.

Method 2 — subtracting the ways being together

1. Write down the number of ways all the objects can be arranged.
2. calculate the number of ways the required objects must be together.
3. Subtract from your answer from step 2 away from your answer to step 1.

E.g. 7 Four girls and two boys, Alan and Tim are to sit in a row. In how many ways can the 6 children be arranged if:

(a) Alan and Tim must sit together

(b) Alan and Tim refuse to sit together

Video: Permutations (items separated)

Solutions to Starter and E.g.s

Exercise

p15 1F Qu 1-3, (4-5 red)

Summary
Items kept together — can be considered one item. The arrangement of the items kept together must also considered

Keeping objects separate:
Method 1 — gap method
4. Write down the number of gaps.
5. Write down the number of ways the separated object can choose the gaps.
6. Write down the number of ways the separated objects can be arranged.
7. Write down the number of ways the non-separated objects can be arranged.
8. Multiply your three answers to step 2, 3 and 4 together.

Method 2 — subtracting the ways being together
1. Write down the number of ways all the objects can be arranged.
2. calculate the number of ways the required objects must be together.
3. Subtract from your answer from step 2 away from your answer to step 1.