Linear vs. angular speed

Starter

1. **(Review of previous material)**
   Convert the following to radians: (a) $35°$ (b) $162.8°$

2. Convert the following to degrees: (a) $\frac{5\pi}{6}$ (b) 2.39^{c}

3. A particle, P, takes 30 seconds to make one revolution of a circle of radius 3 m.
   (a) What angle (in radians) does the particle move through in 1 second?
   (b) What arc length is traversed in 1 second?

Notes

The answer to (a) from the question 3 of the starter is the ***angular speed***, ω (omega) or $\dot{\theta}$. It is measured in rad/s.

N.B. $\dot{\theta} = \frac{d\theta}{dt}$

The answer to (b) is the ***tangential speed*** or ***linear speed.*** It is measured in m/s.

From the A2 mathematics course, arc length, $s = r\theta$ when θ is measured in radians.

$$v = \frac{ds}{dt} = \frac{d(r\theta)}{dt} = r\frac{d\theta}{dt} = r\omega$$ since r is constant during the motion

…or… $v = \dot{s} = r\dot{\theta}$

where $\dot{\theta} = \omega$ is the angular speed in radians per second, rad/s.

So linear velocity = radius × angular velocity

Positive means anticlockwise and negative means clockwise.

E.g. 1 A particle moves in a circle of radius 2 m with tangential speed 3 m/s. Calculate its angular speed.

Working: $v = 3, r = 2$

$$v = r\omega \quad \Rightarrow \quad \omega = \frac{v}{r} = \frac{3}{2} = 1.5 \text{ rad/s}$$

E.g. 2 The tangential speed of an object moving around a circle is given as 8 m/s and the angular velocity is 4 rad/s. Calculate the radius of its path.

E.g. 3 An old gramophone record of diameter 30 cm rotates at 78 revolutions per minute. Calculate its angular speed and the linear speed of the rim in m/s.

Video: Angular speed and accelaration
Video: Angular velocity and speed

Solutions to Starter and E.g.s

Exercise
p84 4A Qu 1-8

Summary
Linear velocity = radius × angular velocity
i.e. $v = r\omega$ where v is the ***tangential speed*** or ***linear speed*** (m/s),
ω (omega) is the ***angular speed*** (rad/s),
and r is the radius of the circle (m)