# Loci in the Argand Diagram

**Starter**

1. **(Review of last lesson)** Convert $1 - \mathrm{i}$ to $[r,\ \theta]$ form.

2. **(Review of last lesson)** Express the complex number $6\ \mathrm{cis}\ \frac{\pi}{3}$ in Cartesian form.

3. Let $z_1 = 5 + 4\mathrm{i}$ and let $z_2 = 1 - \mathrm{i}$.
   (a) Find $|z_1 - z_2|$.
   (b) State what $|z_1 - z_2|$ represents geometrically.

**Notes**

The topic of loci (pronounced "low-sigh") was studied at GCSE — it can also be used on the Argand diagram.

There are three main loci: ***circles***, ***perpendicular bisectors*** and ***half-lines***.

***Distance between points***

$|z_1 - z_2|$ represents the distance between $z_1$ and $z_2$.

***E.g. 1*** (a) Put the equation $|z| = 2$ into words.
(b) Hence describe the loci of all points such that $|z| = 2$.

***Circles***

In words, $|z - z_1| = r$ is "the distance between $z$ and the complex number $z_1$ is $r$".

$|z| = r$ is the circle, centre origin, radius r.
$|z - z_1| = r$ is the circle, centre $z_1$, radius $r$

***E.g. 2*** (a) Put the equation $|z - (4 + 3\mathrm{i})| = 5$ into words.
(b) Hence, on an Argand diagram, draw all the points such that $|z - (4 + 3\mathrm{i})| = 5$.

***Perpendicular bisectors***

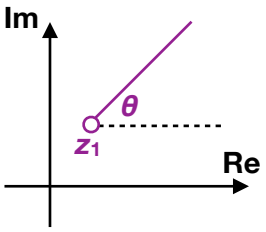
In words, $|z - z_1| = |z - z_2|$ is "the distance between $z$ and the complex number $z_1$ is equal to the distance between $z$ and the complex number $z_2$.

$|z - z_1| = |z - z_2|$ ***is the perpendicular bisector of the line segment joining*** $z_1$ ***and*** $z_2$.

***E.g. 3*** (a) Put the equation $|z - 3| = |z - 5|$ into words.
(b) Hence, on an Argand diagram, draw all the points such that $|z - 3| = |z - 5|$.

***Half-lines***

Arg $(z - z_1) = \theta$ is the ***half-line*** starting at $z_1$ which makes an angle of $\theta$ with the ***real axis***.




Usually the point $z_1$ is not included in the half-line — hence the circle.
If $z_1$ was included in the half-line, the circle would be coloured in.

***E.g. 4*** Describe all the points such that arg $\arg(z - (5 + 2\text{i})) = \frac{\pi}{3}$.

***E.g. 5*** When would the point $z_1$ be included in the half-line?

***Summary***

- $|z_1 - z_2|$ represents the ***distance*** between $z_1$ and $z_2$.
- $|z - z_1| = r$ is the ***circle***, centre $z_1$, radius $r$
- $|z - z_1| = |z - z_2|$ is the ***perpendicular bisector*** of the line segment joining $z_1$ and $z_2$.
- arg $(z - z_1) = \theta$ is the half-line starting at $z_1$ which makes an angle of $\theta$ with the real axis

***N.B.*** For half-lines, $z_1$ is not included in the line (circle) unless arg $z_1 = \theta$

***E.g. 6*** Describe all the points such that:

(a) $|z - 3 + 2\text{i}| = 9$

(b) $\arg(z - (1 + 2\text{i})) = \frac{\pi}{6}$

(c) $|z - (4 + 3\text{i})| = |z + 6 - 2\text{i}|$

(d) $\arg(z - 1 - \text{i}) = \frac{\pi}{4}$

***E.g. 7*** Use an Argand diagram to find, in the form $a + b\text{i}$, the complex number(s) which satisfies

(a) $\arg(z + 1) = \frac{\pi}{4}$ and $\arg(z - 3) = \frac{3\pi}{4}$

(b) $\arg z = \frac{\pi}{6}$ and $|z| = 2$.

***N.B.*** The gradient of a straight line is equal to $\tan\theta$, where $\theta$ is the angle the line makes with the positive $x-$axis.

***Hint:*** draw an Argand diagram.

***Working:*** (a) $\arg(z + 1) = \frac{\pi}{4}$ is the half-line starting at $-1 + 0\text{i}$ at an angle of $\frac{\pi}{4}$ $(45^\circ)$ — this is equivalent to the line $y = x + 1$ for $x > -1$

$\arg(z - 3) = \frac{3\pi}{4}$ is the half-line starting at $3 + 0\text{i}$ at an angle of $\frac{3\pi}{4}$ $(135^\circ)$ — this is equivalent to the line $y = 3 - x$ for $x < 3$

Hence find the point of intersection of $y = x + 1$ and $y = 3 - x$

The point is $(1, 2)$ so $z = 1 + 2\text{i}$

***E.g. 8*** If $\arg z = \frac{\pi}{4}$ and $\arg (z - 3) = \frac{\pi}{2}$, find $\arg (z - 6\text{i})$.

***Hint:*** use an Argand diagram.

Video: Loci of a circle
Video: Loci of perpendicular bisectors
Video: Loci of half-lines
Video: Representing regions in Argand diagrams

Solutions to Starter and E.g.s

**Exercise**
p133 4F Qu 1i, 2i, 3i, 4i, 5-11

***Summary***

- $|z_1 - z_2|$ represents the ***distance*** between $z_1$ and $z_2$.
- $|z - z_1| = r$ is the ***circle***, centre $z_1$, radius $r$
- $|z - z_1| = |z - z_2|$ is the ***perpendicular bisector*** of the line segment joining $z_1$ and $z_2$.
- $\arg (z - z_1) = \theta$ is the half-line starting at $z_1$ which makes an angle of $\theta$ with the real axis