Mean and Variance of Binomial Distribution

Starter

1. (Review of last lesson)
The random variable $W \sim \text{B}(10, 0.35)$. Use your calculator to find:
(a) $P(W = 6)$ (b) $P(W \leq 6)$ (c) $P(W \geq 5)$ (c) $P(4 \leq W \leq 7)$

Notes

The mean and variance of a binomial distribution is easy to calculate
If $X \sim \text{B}(n, p)$ then $P(X = x) = {}^nC_x \times p^x \times (1-p)^{n-x}$
where n is the number of trials
p is the probability of "success"
x is the number of "successes"
$n - x$ is the number of "failures"
$1 - p$ is the probability of "failure"

If $X \sim \text{B}(n, p)$ then
Mean: $\mu = E(X) = np$
Variance: $\sigma^2 = \text{Var}(X) = np(1-p)$
Standard deviation: $\sigma = \sqrt{np(1-p)}$

E.g. 1 If the probability that it is a fine day is 0.4, find the expected number of fine days in a week, and the standard deviation. Explain the problem with the question.

Do not copy

Proof of $\mu = E(X) = np$
Let $X \sim \text{B}(n, p)$ so that the probability distribution is:

x:	0	1	2	3	...	n
$P(X = x)$:	$(1-p)^n$	$np(1-p)^{n-1}$	$\frac{n(n-1)}{2!}p^2(1-p)^{n-2}$	$\frac{n(n-1)(n-2)}{3!}p^3(1-p)^{n-3}$	...	p^n

$$E(X) = \sum_{all\,x} xP(X = x)$$
$$= np(1-p)^{n-1} + \frac{2n(n-1)}{2!}p^2(1-p)^{n-2} + \frac{3n(n-1)(n-2)}{3!}p^3(1-p)^{n-3} + \ldots + np^n$$
$$= np\left[(1-p)^{n-1} + (n-1)p(1-p)^{n-2} + \frac{(n-1)(n-2)}{2!}p^2(1-p)^{n-3} + \ldots + p^{n-1}\right]$$

Let $q = 1 - p$

$$E(X) = np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2!}p^2q^{n-3} + \ldots + p^{n-1}\right]$$
$$= np(q+p)^{n-1}$$

But $q + p = 1$ so $E(X) = np$

E.g. 2 The random variable X is such that $X \sim \text{B}(n, p)$, $E(X) = 2$ and $\text{Var}(X) = \frac{24}{13}$. Find:

(a) the values of n and p and
(b) $P(X = 2)$.

Video: Mean and variance of binomial

Solutions to Starter and E.g.s

Exercise
p30 2D Qu 1i, 2-6

Summary

If $X \sim \text{B}(n, p)$ then

Mean: $\mu = E(X) = np$

Variance: $\sigma^2 = \text{Var}(X) = np(1-p)$

Standard deviation: $\sigma = \sqrt{np(1-p)}$

The mean and variance of a binomial distribution is easy to calculate

If $X \sim \text{B}(n, p)$ then $P(X = x) = {}^nC_x \times p^x \times (1-p)^{n-x}$

where n is the number of trials

p is the probability of "success"

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$n - x$ is the number of "failures"

$1 - p$ is the probability of "failure"