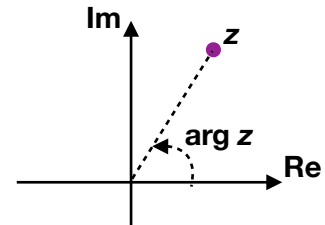


Modulus-Argument Form of a Complex Number

Starter

- (Review of last lesson)**
Convert 240° to an angle in radians, expressing your answer in terms of π .
- (Review of last lesson)** Find the modulus of $4 + 2i$.

N.B. The argument of a complex number, z , is the **angle** that the line **between the origin and z makes with the positive x -axis, measured anti-clockwise**. It is denoted $\arg z$ and is given **in radians**.



- Calculate the argument of the complex numbers:
 - $1 + i$
 - $4i$
 - $-\sqrt{3} + i$

Hint: use an Argand diagram to help you.
- You are given the modulus and argument of a complex number. Express the complex number in the form $x + yi$.
 - Modulus = 6, argument = $\frac{\pi}{3}$
 - Modulus = 2, argument = $\frac{3\pi}{2}$

Hint: draw an Argand diagram to help.

Notes

The modulus-argument form of a complex number consists of the number, r , which is the distance to the origin, and θ , which is the angle the line Oz makes with the **positive x -axis, measured clockwise**.

N.B. $r \geq 0$

The angle θ can take any real value but the principal argument, denoted by $\text{Arg } z$, is defined as $0 \leq \theta < 2\pi$ or $-\pi < \theta \leq \pi$

There are two forms of a complex number:

Cartesian form $x + yi$

Modulus-argument form $[r, \theta]$ – we will see that this notation is rarely used

Converting between Cartesian and modulus-argument forms

Cartesian to modulus-argument form

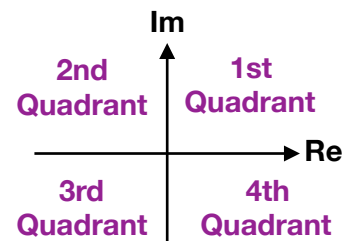
For the complex number $z = x + yi$

The modulus is given by $r = \sqrt{x^2 + y^2}$.

To find the argument:

- Calculate $\tan^{-1} \left| \frac{y}{x} \right|$

N.B. Notice that we ignore the signs of the components of the complex number when finding the initial angle.



2. Sketch a quick Argand diagram to decide which quadrant z lies in and then decide what you need to do to the angle found in step 1.

For example, if z is in the 4th quadrant, subtract the angle found from 2π (since $2\pi \equiv 360^\circ$).

E.g. 1 Convert $-1 - \sqrt{3}i$ to $[r, \theta]$ form.

Working: $r = \left| -1 - \sqrt{3}i \right| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$

Ignore the signs: $\tan^{-1} \frac{\sqrt{3}}{1} = \frac{\pi}{3}$

$-1 - \sqrt{3}i$ is in the 3rd quadrant so we need to add π (180°) to the acute angle

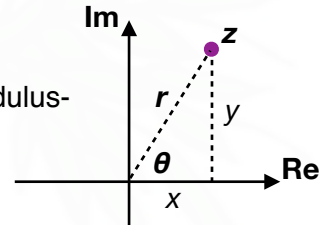
$$\text{Arg}(-1 - \sqrt{3}i) = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$-1 - \sqrt{3}i \equiv \left[2, \frac{4\pi}{3} \right]$$

Modulus-argument to Cartesian form

E.g. 2 For the complex number $z = [r, \theta]$, express the x - and y -coordinates in terms of r and θ . Use the diagram to help you.

This gives us the more common way to express a complex number in modulus-argument form:



$$z = r \cos \theta + i r \sin \theta = r(\cos \theta + i \sin \theta)$$

This is shortened to $z = r \text{cis } \theta$.

E.g. 3 Express the complex number $4 \text{cis } \frac{\pi}{6}$ in Cartesian form.

Video: [Modulus-argument form of complex number](#)

[Solutions to Starter and E.g.s](#)

Exercise

p127 4E Qu 1i, 2i, 3i, 4i, 5i, 6i, 7-11

Summary

Cartesian form

$$x + yi$$

$$x = r \cos \theta \text{ and } y = r \sin \theta$$

Modulus-argument form

$$r \text{cis } \theta \text{ where } r \geq 0 \text{ and } 0 \leq \theta < 2\pi \text{ or } -\pi < \theta \leq \pi$$

Modulus $r = \sqrt{x^2 + y^2}$

Argument

1st quadrant: $\theta = \tan^{-1} \left| \frac{y}{x} \right|$

2nd quadrant: $\theta = \pi - \tan^{-1} \left| \frac{y}{x} \right|$

3rd quadrant: $\theta = \pi + \tan^{-1} \left| \frac{y}{x} \right|$

4th quadrant: $\theta = 2\pi - \tan^{-1} \left| \frac{y}{x} \right|$