Motion in a vertical circle (or conservation of mechanical energy)

Starter

1. **(Review of last lesson)** A circular banked race track has a radius of 200 m. Cars can go round the bend at 44 m/s without side-slipping. Find the angle at which the track is banked.

Notes

So far we have looked at motion with constant velocity in a horizontal circle. Such motion has ***radial acceleration*** $a_r = \dfrac{v^2}{r} = v\omega = r\omega^2$, where $\omega = \dfrac{d\theta}{dt}$, but ***no tangential acceleration***.

We will now look at motion in a vertical circle where the ***tangential*** (or linear) ***velocity*** changes i.e. velocity is variable. Therefore, there is ***tangential acceleration***.

Provided the motion takes place on a smooth track, the velocity can be found at any place in its route using the conservation of energy.

i.e. GPE + KE = constant

- The radial acceleration is the same for motion in a horizontal circle i.e. a_r and it is directed towards the centre. Therefore, use $F = ma_r$ radially.
- For a vertical circle, to complete a full circle both the tension in the string and the speed must be greater than or equal to zero at the highest point.

E.g. 1 A bead is projected with initial velocity 3 m/s from the lowest point of a smooth circular wire of radius 0.2 m fixed in a vertical plane. Find the speed of the bead when it has moved through:

(a) $180°$ (b) $60°$

Working: (a) Initial KE $= \frac{1}{2}m \times 3^2 = 4.5m$

GPE gain $= 0.4mg = 3.92m$

Final KE $= 4.5m - 3.92m = 0.58m$

$\therefore \frac{1}{2}mv^2 = 0.58m \quad \Rightarrow \quad v^2 = 1.16 \quad \Rightarrow \quad v \approx 1.08$

The speed of the bead is 1.08 m/s (3 s.f.)

(b) Initial KE $= \frac{1}{2}m \times 3^2 = 4.5m$

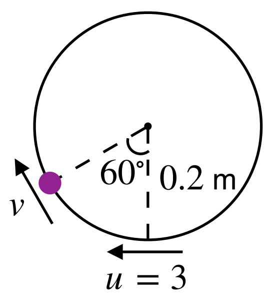
GPE gain $= 0.2mg(1 - \cos 60) = 0.98m$

Final KE $= 4.5m - 0.98m = 3.52m$

$\therefore \frac{1}{2}mv^2 = 3.52m$

$\Rightarrow \quad v^2 = 7.04 \quad \Rightarrow \quad v \approx 2.65$

The speed of the bead is 2.65 m/s (3 s.f.)



E.g. 2 A particle P is fixed to one end of a light rod of length 1.4 m. The other end is smoothly pivoted about O. The rod is released from rest when OP is horizontal. Find the speed of P when OP has rotated through:

(a) $90°$ (b) $60°$.

E.g. 3 Decide whether the beads moving around these smooth vertical circles make a complete revolution. If so, find the velocity at the highest point of the circle. If not, find the height above the centre when it first comes to rest.

(a) Radius $= 1$ m, bead initially at lowest position, initial speed 8 m/s

(b) Radius $= 2$ m, bead initially level horizontally with centre, initial speed 6 m/s downwards

(c) Radius $= 0.5$ m, bead initially $\frac{\pi}{6}$ past the vertical, initial speed 1 m/s downwards

Video: Motion in a vertical circle (string)
Video: Motion in a vertical circle (slack string)

Solutions to Starter and E.g.s

Exercise
p233 9A Qu 1i, 2i, 3-6 (red 7-9)

Summary
For motion in a vertical circle, provided the motion takes place on a smooth track, the velocity can be found at any place in its route using the conservation of energy.

i.e. GPE + KE = constant

- The radial acceleration is the same for motion in a horizontal circle i.e. a_r and it is directed towards the centre. Therefore, use $F = ma_r$ radially.
- For a vertical circle, to complete a full circle both the tension in the string and the speed must be greater than or equal to zero at the highest point.