Permutations with Limited Places

Starter

1. **(Review of last lesson)** Consider the letters of the word PLYMOUTH.
   (a) In how many ways can the letters be arranged?
   (b) How many arrangements begin with two vowels?

2. Twelve children stand in a row. In how many ways can they be arranged if one particular child has to stand at one end?

3. A bookseller is organising her shop window and has 5 books to choose from. She only has room to display 3 books. How many arrangements can she make?

Notes

From the last lesson, the number of permutations (or arrangements) of 5 ***distinct*** objects is $5!$.

$$\text{i.e. } 5 \times 4 \times 3 \times 2 \times 1$$

However, if we can only choose 3 objects there are $5 \times 4 \times 3 = 60$ arrangements

$$\text{i.e. } 5 \times 4 \times 3 = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{5!}{2!} = \frac{5!}{(5-3)!}$$

In general, if there are n ***distinct objects***, the number of ***permutations of*** r of these ***objects*** is:

$$\frac{n!}{(n-r)!}$$

Notation

This is given special notation: $${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

Use the nPr button on your calculator (on the Classwiz: SHIFT >> ×)

E.g. 1 From a group of 15 people, how many arrangements of 6 people can be made?

Working: $${}^{15}P_{6} = \frac{15!}{(15-6)!} = \frac{15!}{9!} = 3603600$$

E.g. 2 In a goal of the month competition, there are 10 candidates. How many different arrangements for the top three positions are there?

E.g. 3 Four different coloured disco lights are to be arranged in a vertical line. How many different arrangements can be made if there are seven different coloured lights to choose from?

E.g. 4 How many five-digit numbers can be formed from 1, 2, 3, 4, 5, 6, 7, 8 and 9 if:
   (a) the digits are all different
   (b) the digits are all the same
   (c) the digits are all different and the number is greater than 60000?

Video: Permutations (limited places)

Solutions to Starter and E.g.s

Exercise
p13 1E Qu 1-9, (10 red)

Summary

If there are n distinct objects, the number of permutations of r of these objects is ${}^{n}P_{r} = \frac{n!}{(n-r)!}$