

## Permuting Objects with Repetitions

### Starter

1. (Review of last lesson)

In how many ways can 5 boys and 4 girls be arranged on a beach if:

- (a) boys and girls are in separate groups
- (b) John, Olivia and Michael want to stay together
- (c) the four girls must be separated
- (d) boys and girls alternate

### Notes

Identical objects mean there are fewer permutations (or arrangements). We *divide* the number of permutations of all the objects by the *number of permutations of the repeated objects*.

So if there are  $n$  objects, of which  $r$  are identical there are  $\frac{n!}{r!}$  permutations.

**E.g. 1** Calculate the permutations of the letters of the word DIGIT.

**Working:** The number of permutations of  $D I_1 G I_2 T$  is  $5!$   
 But  $D I_1 G I_2 T \equiv D I_2 G I_1 T$   
 $I_1$  and  $I_2$  can be arranged in  $2!$  ways *these are identical so divide by  $2!$*   
 Total ways =  $\frac{5!}{2!} = 60$

**E.g. 2** (a) Calculate the arrangements of the letters of the word DEFEATED.  
 (b) If all the E's must be separated, how many permutations of the letters in the word DEFEATED are there?

**E.g. 3** Consider the letters of the word ADDING. How many permutations are there if:

- (a) there are no restrictions
- (b) the 2 D's together
- (c) the 2 D's separated

In general, if there are  $n$  objects with  $r_A$  of type  $A$ ,  $r_B$  of type  $B$ ,  $r_C$  of type  $C$  etc, then the number of permutations is  $\frac{n!}{r_A! \times r_B! \times r_C! \dots}$

**Video:** [Permutations with repeated items](#)

[Solutions to Starter and E.g.s](#)

### Exercise

p16 1G Qu 1-7, (8-10 red)

### Summary

If there are  $n$  objects, of which  $r$  are identical, there are  $\frac{n!}{r!}$  permutations.

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