Probability distributions (AS Ma)

Starter

1. Use your calculator to find the mean and standard deviation of the data set:
   28, 41, 18, 24, 37, 52, 33, 35.

Notes

Definitions

A ***random variable*** is a variable whose sum of possible outcomes equals 1.

A ***discrete random variable*** is a random variable whose outcomes can only take a ***countable*** number of values.

N.B. Discrete is opposite to continuous.

Further examples of discrete and continuous random variables

Discrete random variables	Continuous random variables
Flipping a coin	Height/weight of people
Rolling a dice	Time taken to complete a task

Probability function and probability distribution

A ***probability density function*** (pdf) describes how probabilities are assigned.
A ***probability distribution*** is the set of all possible values of a random variable together with associated probabilities. This is usually expressed in a table.
CAPITAL LETTERS are used to describe random variables and ***lower case letters*** are used to represent the values that the random variable can take.

For example, for a 6-sided dice:

Probability function: $P(X = x) = \frac{1}{6}$ where $x = 1,2,3,...,6$

Probability distribution:

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

N.B. The sum of the probabilities is 1 — this shows it is a random variable.

E.g. 1 A tetrahedral die has the numbers 1, 2, 3 and 4 on its faces. The die is biased in such a way that the probability of the die landing on the number x is inversely proportional to x.

For example, $P(X = 3) = \frac{k}{3}$ where k is a constant.

(a) Write down the probability density function in terms of k.

(b) Find the probability distribution for X, the number the die lands on after a single roll.

Working: (a) $P(X = x) = \frac{k}{x}$ where $x = 1,2,3,4$

(b) The sum of the probabilities is 1 so $\frac{k}{1} + \frac{k}{2} + \frac{k}{3} + \frac{k}{4} = 1$

so $k = \frac{12}{25}$

x	1	2	3	4
$P(X = x)$	$\frac{12}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{3}{25}$

E.g. 2 Let X be the discrete variable 'the number of fours obtained when two dices are thrown'.

(a) Find the probability distribution.

(b) Show that X is random variable.

Video: Probability distribution tables

Solutions to Starter, E.g.s and Exercise

Exercise

Answers can be found via the blue link above.

1. A discrete random variable X has the following probability distribution:

x :	1	2	3	4	
$P(X = x)$:	$\frac{1}{3}$	$\frac{1}{3}$	k	$\frac{1}{4}$	where k is a constant

(a) Find the value of k.

(b) Find $P(X \leq 3)$.

2. The probability density function of a discrete random variable is given by $P(X = x) = kx$ for $x = 12, 13, 14$. Find the value of the constant k.

3. The pdf of a discrete random variable Y is given by $P(Y = y) = cy^2$ for $y = 0, 1, 2, 3, 4$. Given that c is a constant find its value.

4. Two tetrahedral dice, each with faces labelled 1, 2, 3 and 4 are thrown and the score noted, where the score is the sum of the two numbers. If X is the random variable 'the score when two tetrahedral dice are thrown:

(a) Find the probability distribution of X.

(b) Find the probability density function of X.

5. A drawer contains 8 brown and 4 blue socks. A sock is taken from the drawer at random, its colour is noted and it is then replaced. The procedure is done three times. If X is the random variable 'the number of brown socks taken', find the probability distribution for X.

Summary

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