

Sum and Product of Roots (Quadratics)

Starter

1. **(Review of last lesson)**
The equation $z^4 - 4z^3 + 3z^2 + 2z - 6 = 0$ has a root $1 - i$. Find the other roots.
2. The quadratic formula gives the two solutions of the equation $ax^2 + bx + c = 0$ as $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.
Find: (a) the sum of the roots (b) the product of the roots.
3. Using your answers to question 2, **write down** the sum and product of the roots of the quadratic equation $12x^2 + 11x - 56 = 0$.

Notes

For the quadratic equation $ax^2 + bx + c = 0$, let the roots be alpha (α) and beta (β). Therefore:

$$\text{Sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a}$$

E.g. 1 Find the simplest quadratic equation with the roots 2 and 3.

Working: The simplest cubic has 1 as the coefficient of x^3 .
Sum of roots = 5 **(change the sign)**
Product of roots = 6 **(don't change the sign)**
So simplest equation is $x^2 - 5x + 6 = 0$.

E.g. 2 Write down a quadratic equation with integer coefficients such that the sum of the roots is $\frac{1}{2}$ and the product is 3.

E.g. 3 Write down the expansions of: (a) $(\alpha + \beta)^2$ (b) $(\alpha - \beta)^2$

Working: (a) $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$
(b) $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

E.g. 4 Let the roots of the equation $x^2 - 4x + 2 = 0$ be α and β . Without finding α and β , find the values of the following:

- (a) $3\alpha + 3\beta$ (b) $\alpha^2 + 2\alpha\beta + \beta^2$ (c) $\alpha^2 - \alpha\beta + \beta^2$
(d) $(\alpha - \beta)^2$ (e) $\frac{1}{\alpha} + \frac{1}{\beta}$ (f) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

Hint: find $\alpha + \beta$ and $\alpha\beta$ then express each question in terms of them.
For (e) and (f), add the algebraic fractions.

Working:

(a) Sum of roots = $\alpha + \beta = 4$
Product of roots = $\alpha\beta = 2$
 $3\alpha + 3\beta = 3(\alpha + \beta) = 3 \times 4 = 12$

(b) $\alpha^2 + 2\alpha\beta + \beta^2 = (\alpha + \beta)^2 = 4^2 = 16$

(c) $\alpha^2 - \alpha\beta + \beta^2 = \alpha^2 + \beta^2 - \alpha\beta$
 $= (\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta$
 $= 4^2 - 3 \times 2$
 $= 10$

(d) $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$ *this now looks similar to (c)*
 $= \alpha^2 - \alpha\beta + \beta^2 - \alpha\beta$
 $= 10 - 2$
 $= 8$

(e) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta}{\alpha\beta} + \frac{\alpha}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{4}{2} = 2$

(f) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2}{\alpha^2\beta^2} + \frac{\alpha^2}{\alpha^2\beta^2}$
 $= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$
 $= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$
 $= \frac{4^2 - 2 \times 2}{2^2}$
 $= 3$

Video: [Sum and product of roots \(quadratic\)](#)

[Solutions to Starter and E.g.s](#)

Exercise

p150 5C Qu 1, 2

Summary

For the quadratic equation $ax^2 + bx + c = 0$:

Sum of roots = $\alpha + \beta = -\frac{b}{a}$
Product of roots = $\alpha\beta = \frac{c}{a}$