

Topic X1 Matrices (Post-TT B) [40]

1.

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$, where a is a constant.

(i) Find \mathbf{A}^{-1} . [2]

The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$.

(ii) Given that $\mathbf{PA} = \mathbf{B}$, find the matrix **P**. [3]

2.

The transformation **S** is a shear with the y -axis invariant (i.e. a shear parallel to the y -axis). It is given that the image of the point (1, 1) is the point (1, 0).

(i) Draw a diagram showing the image of the unit square under the transformation **S**. [2]

(ii) Write down the matrix that represents **S**. [2]

3.

Given that **A** and **B** are non-singular square matrices, simplify

$$\mathbf{AB}(\mathbf{A}^{-1}\mathbf{B})^{-1}. \quad [3]$$

4.

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$.

(i) Given that **A** is singular, find a . [2]

(ii) Given instead that **A** is non-singular, find \mathbf{A}^{-1} and hence solve the simultaneous equations

$$\begin{aligned} ax + 3y &= 1, \\ -2x + y &= -1. \end{aligned} \quad [5]$$

5.

(i) The transformation **P** is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Give a geometrical description of transformation **P**. [2]

(ii) The transformation **Q** is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Give a geometrical description of transformation **Q**. [2]

(iii) The transformation **R** is equivalent to transformation **P** followed by transformation **Q**. Find the matrix that represents **R**. [2]

(iv) Give a geometrical description of the **single** transformation that is represented by your answer to part (iii). [3]

6.

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}$.

(i) Find the value of a for which **A** is singular. [5]

(ii) Given that **A** is non-singular, find \mathbf{A}^{-1} and hence solve the equations

$$\begin{aligned} ax + 2y + z &= 1, \\ x + 3y + 2z &= 2, \\ 4x + y + z &= 3. \end{aligned} \quad [7]$$