

Topic X1 Matrices (Pre-TT A) [44]

1.

The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$ and **I** is the 2×2 identity matrix.
Find the values of the constants a and b for which $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$. [4]

(Total 4 marks)

2.

The transformation **S** is a shear parallel to the x -axis in which the image of the point $(1, 1)$ is the point $(0, 1)$.

(i) Draw a diagram showing the image of the unit square under **S**. [2]

(ii) Write down the matrix that represents **S**. [2]

(Total 4 marks)

3.

S is a singular matrix such that

$$\det \mathbf{S} = \begin{vmatrix} a & a & x \\ x-b & a-b & x+1 \\ x^2 & a^2 & ax \end{vmatrix}$$

Express the possible values of x in terms of a and b .

[7 marks]

4.

The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ -2 & a \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 7 & 3 \\ 1 & 5 \end{pmatrix}$, where $a \neq -8$ and **I** is the 2×2 identity matrix. Find

(i) $7\mathbf{A} - \mathbf{I}$, [2]

(ii) $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1}$. [3]

(Total 5 marks)

5.

The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$.

(i) Find \mathbf{C}^{-1} . [2]

(ii) Given that $\mathbf{C} = \mathbf{AB}$, where $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, find \mathbf{B}^{-1} . [5]

(Total 7 marks)

6.

A company runs three theme parks, A (Aztec Adventureland), B (Babylonian Towers) and C (Carthaginian Kingdom).

It is known that park A makes a profit of £30 per visitor, park B makes a profit of £26 per visitor and park C makes a profit of £33 per visitor.

In 2017 the Aztec Adventureland park was upgraded, which took one year to carry out. During 2017

- park A had only 50% of the number of visitors it had in 2016
- park B had 25% more than the number of visitors it had in 2016
- park C had 15% more than the number of visitors it had in 2016

In total 1 350 000 people visited the three theme parks during 2017.

The company made a total profit from the parks of £39.15 million in 2016. The profits dropped by 1% for 2017.

Form and solve a matrix equation to find, to 2 significant figures, the number of visitors for each of the theme parks in 2016.

(8)

(Total 8 marks)

7.

$$\mathbf{P} = \begin{pmatrix} 3 & 3 \\ 4 & 7 \end{pmatrix}$$

The matrix \mathbf{P} represents a linear transformation, T , of the plane.

(a) Describe the invariant points of the transformation T .

(3)

(b) Describe the invariant lines of the transformation T .

(6)

(Total 9 marks)