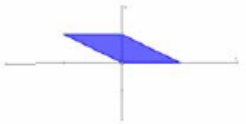
Topic X1 Matrices (Pre-TT A) [44] MARKSCHEME

1.

$3a+5b=1,\ a+2b=1$	M1		Obtain a pair of simultaneous equations
	M1		
$a=-3,\ b=2$	A1 A1	4	Attempt to solve
		4	Obtain correct answers.

2.

(i) 	M1		For 2 other correct vertices
	A1	2	For completely correct diagram
(ii) $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$	B1 B1	2	Each column correct
		4	

3.

Explains that det**M**=0 when **M** is singular (Seen anywhere)	AO2.4	R1	**S** is singular $\Rightarrow \begin{vmatrix} a & a & x \\ x-b & a-b & x+1 \\ x^2 & a^2 & ax \end{vmatrix}=0$
Seeks factor by combining rows or columns to find a first linear factor for example $C_1'=C_1-C_2$	AO3.1a	M1	$\det \mathbf{S}=\begin{vmatrix} 0 & a & x \\ x-a & a-b & x+1 \\ x^2-a^2 & a^2 & ax \end{vmatrix}$
Extracts first factor correctly	AO1.1b	A1	$=(x-a)\begin{vmatrix} 0 & a & x \\ 1 & a-b & x+1 \\ x+a & a^2 & ax \end{vmatrix}$
Combines rows or columns to find a second linear factor $R_3'=R_3-aR_1$	AO1.1a	M1	$\det \mathbf{S}=(x-a)\begin{vmatrix} 0 & a & x \\ 1 & a-b & x+1 \\ x+a & 0 & 0 \end{vmatrix}$
Extracts second factor correctly	AO1.1b	A1	$=(x-a)(x+a)\begin{vmatrix} a & x \\ a-b & x+1 \end{vmatrix}$
Completes expansion and obtains final factor	AO1.1b	A1	$=(x-a)(x+a)(a+bx)$
Deduces correct values of x FT 'their' factors	AO2.2a	A1F	$(x-a)(x+a)(a+bx)=0$ $x=a,-a,-\frac{a}{b}$

4.

(i)		$7\mathbf{A}-\mathbf{I}=\begin{pmatrix}6 & 28\\ -14 & 7a-1\end{pmatrix}$	B2 [2]	B1 for 3 elements correct or B1 for 4 elements correct but brackets omitted
(ii)		*Either* $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1}=\mathbf{BA}$ $\begin{pmatrix}1 & 28+3a\\ -9 & 4+5a\end{pmatrix}$ *Or* $\mathbf{A}^{-1}=\frac{1}{a+8}\begin{pmatrix}a & -4\\ 2 & 1\end{pmatrix}$, $\mathbf{B}^{-1}=\frac{1}{82}\begin{pmatrix}5 & -3\\ -1 & 7\end{pmatrix}$ $\begin{pmatrix}1 & 28+3a\\ -9 & 4+5a\end{pmatrix}$	B1 M1 A1 [3] B1 M1 A1	Stated or used Attempt at multiplication of **BA** or **AB**, 2 elements correct Obtain correct answer Both correct Attempt at multiplication of their $\mathbf{A}^{-1}\mathbf{B}^{-1}$ only, ignore dets, 2 elements correct Obtain correct answer

5.

(i) $\frac{1}{2}\begin{pmatrix}8 & -2\\ -3 & 1\end{pmatrix}$	B1 B1		Transpose leading diagonal and negate other diagonal Divide by determinant
(ii) Either $\frac{1}{2}\begin{pmatrix}14 & 2\\ -5 & 0\end{pmatrix}$ Or $\frac{1}{5}\begin{pmatrix}3 & -1\\ -1 & 2\end{pmatrix}$ $\mathbf{B}=\mathbf{A}^{-1}\mathbf{C}$ $\mathbf{B}=\frac{1}{5}\begin{pmatrix}0 & -2\\ 5 & 14\end{pmatrix}$ $\frac{1}{2}\begin{pmatrix}14 & 2\\ -5 & 0\end{pmatrix}$ Or $\mathbf{AB}=\begin{pmatrix}2a+c & 2b+d\\ a+3c & b+3d\end{pmatrix}$ $a=0, c=1, b=-0.4, d=2.8$ $\frac{1}{2}\begin{pmatrix}14 & 2\\ -5 & 0\end{pmatrix}$	B1 M1 A1ft B1 M1 M1 A1ft A1 B1 M1 A1A1 A1	2 5 7	State or imply $(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$ Use this result and obtain $\mathbf{B}^{-1}=\mathbf{C}^{-1}\mathbf{A}$, or equivalent matrix algebra Matrix multn., two elements correct, for any pair All elements correct ft their (i) Find $\mathbf{A}^{-1}$ Premultiply by $\mathbf{A}^{-1}$ stated or implied Matrix multn. Two elements correct All elements correct Correct $\mathbf{B}^{-1}$ Find **AB** Solve one pair of simultaneous equations Each pair of answers Correct $\mathbf{B}^{-1}$

6.

2	Profit in 2017 is 0.99×£39.15m = £38.7585m	B1	2.2a
	Let x = number of visitors to park A in 2016, y = number of visitors to park B in 2016 and z = number of visitors to park C in 2016. So $0.5x + 1.25y + 1.15z = 1.35\times10^6$ $30x + 26y + 33z = 39.15\times10^6$ $15x + 32.5y + 37.95z = 38.7585\times10^6$	M1 A1	3.1b 1.1b

	Hence $\begin{pmatrix}0.5 & 1.25 & 1.15\\ 30 & 26 & 33\\ 15 & 32.5 & 37.95\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1.35\times10^6\\39.15\times10^6\\38.7585\times10^6\end{pmatrix}$	M1 A1	3.1a 1.1b
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0.5 & 1.25 & 1.15\\ 30 & 26 & 33\\ 15 & 32.5 & 37.95\end{pmatrix}^{-1}\begin{pmatrix}1.35\times10^6\\39.15\times10^6\\38.7585\times10^6\end{pmatrix}=\begin{pmatrix}\\\\\end{pmatrix}$	M1	1.1b
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}400331.75...\\593257.41...\\355010.74...\end{pmatrix}$	A1	1.1b
	So in 2016 park *A* had 400 000 visitors, park *B* had 590 000 visitors and park *C* had 360 000 visitors to 2 s.f.	A1ft	3.2a
Note	$\begin{pmatrix}0.5 & 1.25 & 1.15\\ 30 & 26 & 33\\ 15 & 32.5 & 37.95\end{pmatrix}^{-1}=\begin{pmatrix}0.4916... & 0.0576... & -0.0650...\\ 3.6871... & -0.0098... & -0.1031...\\ -3.3519... & -0.0143... & 0.1403...\end{pmatrix}$		

7.

3(a)	$\begin{pmatrix}3 & 3\\4 & 7\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}\Rightarrow\begin{matrix}3x+3y=x\\4x+7y=y\end{matrix}$	M1	1.1b
	$\Rightarrow\begin{matrix}2x=-3y\\4x=-6y\end{matrix}\Rightarrow y=-\frac{2}{3}x$	M1	1.1b
	So the invariant points of the transformation are precisely those points lying on the line $y=-\frac{2}{3}x$.	A1	2.4
		(3)	
(b)	$\begin{pmatrix}3 & 3\\4 & 7\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}x'\\mx'+c\end{pmatrix}\Rightarrow\begin{matrix}3x+3(mx+c)=x'\\4x+7(mx+c)=mx'+c\end{matrix}$	M1	3.1a
	$\Rightarrow 4x+7(mx+c)=m(3x+3mx+3c)+c\Rightarrow(..)x+..=..$	M1	1.1b
	$\Rightarrow(4+7m-3m-3m^2)x+7c=3mc+c$ (oe)	A1	1.1b
	$\Rightarrow(2-m)(2+3m)x+3c(2-m)=0\Rightarrow(2-m)\big((2+3m)x+3c\big)=0$ $\Rightarrow m-2=0$ or both $2+3m=0$ and $c=0$ (since the equation must hold for all *x* to give fixed lines)	M1	3.1a
	Since the equations are satisfied whenever $m=2$, the lines $y=2x+c$ are invariant lines under *T*.	A1	2.4
	Also, as the equation holds when $m=-2/3$ and $c=0$, the line $y=-\frac{2}{3}x$ is invariant – or notes that this line is invariant as all the points on it are invariant as shown in (a).	B1	2.2a
		(6)	
			(9 marks)