

## Topic X1 Matrices (Pre-TT B) [43]

1.

The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} a & -1 \\ -3 & -2 \end{pmatrix}$ .

(i) Given that  $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$ , write down the value of  $a$ . [1]

(ii) Given instead that  $\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$ , find the value of  $a$ . [2]

(Total 3 marks)

2.

Given that **A** and **B** are  $2 \times 2$  non-singular matrices and **I** is the  $2 \times 2$  identity matrix, simplify

$$\mathbf{B}(\mathbf{AB})^{-1}\mathbf{A} - \mathbf{I}. \quad [4]$$

(Total 4 marks)

3.

The matrix **M** is given by  $\mathbf{M} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ .

(i) Find the value of the determinant of **M**. [3]

(ii) State, giving a brief reason, whether **M** is singular or non-singular. [1]

(Total 4 marks)

4.

The matrix  $\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$  represents the transformation **P**.

(i) Describe fully the transformation **P**. [3]

Transformation **Q** is a stretch, parallel to the  $y$ -axis with scale factor 4.

(ii) Find the matrix that represents transformation **Q**. [2]

Transformation **T** is equivalent to transformation **P** followed by transformation **Q**.

(iii) Find the matrix that represents transformation **T**. [2]

(iv) Find the area of the image of the unit square under transformation **T**. [2]

(Total 9 marks)

5.

By using the determinant of an appropriate matrix, find the values of  $k$  for which the simultaneous equations

$$kx + 8y = 1,$$

$$2x + ky = 3,$$

do not have a unique solution.

[3]

(Total 3 marks)

6.

The matrix  $C$  is given by  $C = \begin{pmatrix} 0 & 3 \\ -1 & 0 \end{pmatrix}$ .

- (i) Draw a diagram showing the unit square and its image under the transformation represented by  $C$ . [2]

The transformation represented by  $C$  is equivalent to a rotation,  $R$ , followed by another transformation,  $S$ .

- (ii) Describe fully the rotation  $R$  and write down the matrix that represents  $R$ . [3]  
(iii) Describe fully the transformation  $S$  and write down the matrix that represents  $S$ . [4]

(Total 9 marks)

7.

The matrix  $D$  is given by  $D = \begin{pmatrix} a & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}$ , where  $a \neq 2$ .

- (i) Find  $D^{-1}$ . [7]  
(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} ax + 2y &= 3, \\ 3x + y + 2z &= 4, \\ -y + z &= 1. \end{aligned} \quad [4]$$

(Total 11 marks)