Topic X2 Vectors and induction (Post-TT A) [40] MARKSCHEME

1.

(i)	$\mathbf{n} = [2, 1, 3] \times [3, 1, 5]$	M1	For using direction vectors and attempt to find vector product
	$= [2, -1, -1]$	A1 **2**	For correct direction (allow multiples)
(ii)	$d = \dfrac{\lvert [5, 2, 1] . [2, -1, -1] \rvert}{\sqrt{6}}$	B1	For $(\mathbf{AB} =) [5, 2, 1]$ or any vector joining lines
		M1	For attempt at evaluating $\mathbf{AB} . \mathbf{n}$
		M1	For $\lvert \mathbf{n} \rvert$ in denominator
	$= \dfrac{7}{\sqrt{6}} = \tfrac{7}{6}\sqrt{6} = 2.8577$	A1 **4**	For correct distance
		6	

2.

Uses vector product and expands brackets correctly	AO1.1a	M1	$\lvert (\mathbf{a}+5\mathbf{b})\times(\mathbf{a}-4\mathbf{b}) \rvert$
Uses the correct notation and correct order with the vector product.	AO2.5	B1	$= \lvert \mathbf{a}\times\mathbf{a} - 4\mathbf{a}\times\mathbf{b} + 5\mathbf{b}\times\mathbf{a} - 20\mathbf{b}\times\mathbf{b} \rvert$ $= \lvert \mathbf{0} - 4\mathbf{a}\times\mathbf{b} + 5\mathbf{b}\times\mathbf{a} - \mathbf{0} \rvert$
Reduces the number of terms in 'their' expression by using $\mathbf{a}\times\mathbf{a} = \mathbf{b}\times\mathbf{b} = \mathbf{0}$	AO1.1a	M1	since **a** is parallel to **a** and **b** is parallel to **b** then $\mathbf{a}\times\mathbf{a} = \mathbf{0}$ and $\mathbf{b}\times\mathbf{b} = \mathbf{0}$ $= \lvert -4\mathbf{a}\times\mathbf{b} - 5\mathbf{a}\times\mathbf{b} \rvert$ since $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$
and explains their reasoning (must have clear statement that $\mathbf{a}\times\mathbf{a} = \mathbf{0}$)	AO2.4	E1	$= \lvert -9\mathbf{a}\times\mathbf{b} \rvert$
Uses $-\mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{a}$ to collect 'their' terms together	AO1.1a	M1	$= 9\lvert \mathbf{a}\times\mathbf{b} \rvert$ $= 9\lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert \sin 90$
and explains their reasoning (must have clear statement that $-\mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{a}$ OE)	AO2.4	E1	$= 9\lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert$
Recalls correctly the formula for the modulus of the vector product (may see $\lvert \mathbf{a} \rvert \times \lvert \mathbf{b} \rvert \sin\theta$ or may see $\lvert \mathbf{a} \rvert \times \lvert \mathbf{b} \rvert \sin 90^\circ$)	AO1.2	B1	
Obtains $\lvert \mathbf{a}\times\mathbf{b} \rvert = \lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert$ since vectors **a** and **b** are perpendicular	AO1.1b	A1	
Compares expressions to correctly deduce the value of k CAO	AO2.2a	R1	Hence $k = 9$

3.

3(i)	When n = 1, LHS = $\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}$, RHS = $\begin{pmatrix} 3\times1+1 & -1 \\ 9\times1 & 1-3\times1 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}$. So the statement is true for n = 1	B1	2.2a
	Assume true for n = k, so $\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^k = \begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}$	M1	2.4
	Then $\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^{k+1} = \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}\begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}$ or $\begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}$	M1	2.1
	$= \begin{pmatrix} 4(3k+1)-9k & -4k-(1-3k) \\ 9(3k+1)-18k & -9k-2(1-3k) \end{pmatrix}$ or $\begin{pmatrix} 4(3k+1)-9k & -(3k+1)+2k \\ 36k+9(1-3k) & -9k-2(1-3k) \end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix} 3(k+1)+1 & -(k+1) \\ 9(k+1) & 1-3(k+1) \end{pmatrix}$	A1	1.1b
	Hence the result is true for n = k+1. Since it is <u>true for n = 1</u>, and <u>if true for k = n then true for n = k+1</u>, thus by mathematical induction the <u>result holds for all n ∈</u>	A1 cso	2.4
		(6)	
(ii)	(a) $2^2 = 4 \not< 4 = 2^2$ OR $3^2 = 9 \not< 8 = 2^3$ OR $4^2 = 16 \not< 16 = 2^4$	B1	1.1b
	(b) The statement $2k+1 < k^2$ is not true for all positive integers.	B1	1.1b
	(c) The statement in line 4 is true for positive integers $k > 2$ so the induction hypothesis is true for $n > 2$. So the induction holds from any base case greater than 2.	M1	2.3
	Since the result is true for n = 5 as $5^2 = 25 < 32 = 2^5$ and $2k + 1 < k^2$ also true for $k > 5$ so the induction holds with base case n = 5.	A1	2.4
	But not true for n = 2, 3 or 4 as $2^2 = 4 \not< 4 = 2^2$ and $3^2 = 9 \not< 8 = 2^3$ and $4^2 = 16 \not< 16 = 2^4$. Hence true for n = 1 and for n Ö 5	A1	2.1
		(5)	
			(11 marks)

Notes:

(a)

B1: Shows the general form holds for n = 1.

M1: Makes the inductive assumption, assume true for n = k.

M1: Attempts the multiplication either way.

A1: Correct matrix in terms of k.

A1: Rearranged into correct form to show true for k + 1.

A1: Completes the inductive argument conveying **all** three underlined points or equivalent at some point in their argument.

(b)(i)

B1: Provides a suitable counter example using n = 2, 3 or 4. Accept = in place of $\nless$ as long as there is a suitable conclusion with it.

(b)(ii)

B1: Identifies the error as in the scheme or equivalent (e.g. $k^2 + 2k + 1 < 2k^2$ is not always true).

(b)(iii)

M1: Identifies that the induction is valid as long as $2k+1 < k^2$ is true which happens for kÖ3 (accept any value greater than 3 for this mark).

A1: Correct base case of 5 and explains the proof given holds for integers greater than or equal to 5.

A1: Complete argument correct. All positive integers satisfying the inequality identified, with demonstration that 2, 3 and 4 do not.

4.

(i)		If MR, mark according to the scheme & follow-through from candidate's data. Award M, A & B marks (where possible) & apply penalty of 1 mark (by withholding one A mark in the question). E.g. in (i), product to be 'correct' & 'not perpendicular' to be stated. α. Full justification that $t = -1$. May be 'by inspection'. [No equations not satisfied by $t = -1$ to be shown] ['unusual' attempts must be carefully checked; if convinced, award the B1 e.g. displacement vector between (-3**i** + 6**k**) and (-**i** + 2**j** + 7**k**) = ±(2**i** + 2**j** + **k**)] β. Consider scalar product $\begin{pmatrix}-3\\0\\6\end{pmatrix}.\begin{pmatrix}2\\2\\1\end{pmatrix}$ Show – **6** + **(0)** + **6** = 0 and somewhere state perpendicularity oe [If $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ quoted, ignore accuracy of work involving $\lvert\mathbf{a}\rvert$ and $\lvert\mathbf{b}\rvert$]	B1 M1 A1 [3]	No other t = to be mentioned
(ii)		Use **r** = v (-3**i** +6**k**) and ℓ_2 Attempt to produce at least two relevant equations Solve two equations & produce $(v, s) = (\frac{1}{3}, -3)$ soi Demonstrate clearly that these satisfy third equation	*M1 M1dep* A1 B1 [4]	or $(-3\mathbf{i}+6\mathbf{k}) + v(-3\mathbf{i}+6\mathbf{k})$ $(v, s) = (-\frac{2}{3}, -3)$ Numerical proof required
(iii)		Method for finding $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{OA}\rvert$ or $\lvert\overrightarrow{AB}\rvert$ $\lvert\overrightarrow{OB}\rvert = \sqrt{5}$ or $\lvert\overrightarrow{OA}\rvert = \sqrt{45}$ oe or $\lvert\overrightarrow{BA}\rvert = \sqrt{20}$ oe Obtain 3:2 oe	M1 A1 A1 [3]	Method for finding $\overrightarrow{OB}$ or $\overrightarrow{BO}$ or $\overrightarrow{AB}$ or $\overrightarrow{BA}$ $\overrightarrow{OB} = \begin{pmatrix}-1\\0\\2\end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix}-2\\0\\4\end{pmatrix}$ Answer 3:2 WW → B3

5.

(i) Working out **b – a** or **a – b** or **c – a** or **a – c**	**M1**	)	Irrespective of label
= ± (– 3**i** – **j** – **k**) or ± (– 2**i** + **j** – 2**k**)	**A1**	)	If not scored ,these 1st 3 marks can be
Method for finding magnitude of any vector	**M1**	)	awarded in part (ii)
Method for finding scalar product of any 2 vectors	**M1**		
Using $\cos\theta = \frac{a.b}{\lvert a\rvert\lvert b\rvert}$ AEF for any 2 vectors	**M1**		
[Alternative cosine rule method $\lvert\overrightarrow{BC}\rvert = \sqrt{6}$	**B1**		
Cosine rule used	**M1**		'Recognisable' form
45.3°, 0.79(0), $\frac{\pi}{3.97}$ (45.289378, 0.7904487)	**A1**	6	Do not accept supplement (134.7 etc)

(ii) Use of $\frac{1}{2}\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert\sin\theta$	**M1**		Accept $\lvert\frac{1}{2}\overrightarrow{AB} \times \overrightarrow{AC}\rvert$
3.54 (3.5355) or $\frac{5\sqrt{2}}{2}$	**A1**	2	Accept from correct supp (134.7 etc)