

Topic X2 Vectors and induction (Post-TT B) [42]

1.

Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. [3]

(Total 3 marks)

2.

The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Find \mathbf{M}^2 and \mathbf{M}^3 . [3]

(ii) Hence suggest a suitable form for the matrix \mathbf{M}^n . [1]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

(iv) Describe fully the single geometrical transformation represented by \mathbf{M}^{10} . [3]

(Total 11 marks)

3.

Lines L_1 , L_2 and L_3 have vector equations

$$L_1: \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}),$$

$$L_2: \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}),$$

$$L_3: \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + u(3\mathbf{i} + c\mathbf{j} + \mathbf{k}).$$

(i) Calculate the acute angle between L_1 and L_2 . [4]

(ii) Given that L_1 and L_3 are parallel, find the value of c . [2]

(iii) Given instead that L_2 and L_3 intersect, find the value of c . [5]

(Total 11 marks)

4.

Prove that $8^n - 7n + 6$ is divisible by 7 for all integers $n \geq 0$

[5 marks]

5.

$ABCD$ is a parallelogram. The position vectors of A , B and C are given respectively by

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j}, \quad \mathbf{c} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}.$$

(i) Find the position vector of D . [3]

(ii) Determine, to the nearest degree, the angle ABC . [4]

(Total 7 marks)

6.

Relative to an origin O , the points A and B have position vectors $3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ respectively.

(i) Find a vector equation of the line passing through A and B . [2]

(ii) Find the position vector of the point P on AB such that OP is perpendicular to AB . [5]

(Total 7 marks)