Topic X2 Vectors and induction (Post-TT B) [44] MARKSCHEME

1.

$2^2+(-3)^2+(\sqrt{12})^2$ soi e.g. 25 or 5	M1	Allow $2^2-3^2+\sqrt{12}^2$
5	A1	May be implied by 5 or 1/5 in final answer
$\frac{1}{5}\begin{pmatrix}2\\-3\\\sqrt{12}\end{pmatrix}$ or $\begin{pmatrix}\frac{2}{5}\\-\frac{3}{5}\\\frac{\sqrt{12}}{5}\end{pmatrix}$ AEF	√A1 3	FT their '5'. Accept $-\frac{1}{5}\begin{pmatrix}\\\\\end{pmatrix}$ or $\frac{1}{\pm 5}\begin{pmatrix}\\\\\end{pmatrix}$

[3]

2.

(i) $\mathbf{M}^2=\begin{pmatrix}1&4\\0&1\end{pmatrix}$ $\mathbf{M}^3=\begin{pmatrix}1&6\\0&1\end{pmatrix}$	B1		Correct $\mathbf{M}^2$ seen
	M1		Convincing attempt at matrix multiplication for $\mathbf{M}^3$
	A1	3	Obtain correct answer
(ii) $\mathbf{M}^n=\begin{pmatrix}1&2n\\0&1\end{pmatrix}$	B1ft	1	State correct form, consistent with (i)
(iii)	M1		Correct attempt to multiply $\mathbf{M}$ & $\mathbf{M}^k$ or v.v.
	A1		Obtain element $2(k+1)$
	A1		Clear statement of induction step, from correct working
	B1	4	Clear statement of induction conclusion, following their working
(iv)	B1		Shear
	DB1		x-axis invariant
	DB1	3	e.g. $(1, 1) \rightarrow (21, 1)$ or equivalent using scale factor or angles

[11]

3.

(i) Use $-6\mathbf{i}+8\mathbf{j}-2\mathbf{k}$ and $\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ only	M1		
Correct method for scalar product	M1		of any two vectors $(-6+24-4=14)$
Correct method for magnitude	M1		of any vector $(\sqrt{36+64+4}=\sqrt{104}$ or $\sqrt{1+9+4}=\sqrt{14})$
68 or 68.5 (68.47546); 1.2(0) (1.1951222) rad	A1	4	
[N.B. 61 (60.562) will probably have been generated by $5\mathbf{i}+\mathbf{j}-2\mathbf{k}$ and $3\mathbf{i}-8\mathbf{j}$]			
(ii) Indication that relevant vectors are parallel	M1		$-6\mathbf{i}+8\mathbf{j}-2\mathbf{k}$ & $3\mathbf{i}+c\mathbf{j}+\mathbf{k}$ with some indic of method of attack eg $-6\mathbf{i}+8\mathbf{j}-2\mathbf{k}=\lambda(3\mathbf{i}+c\mathbf{j}+\mathbf{k})$
c = −4	A1	2	c = −4 WW → B2
(iii) Produce 2/3 equations containing t,u (& c)	M1		eg $3+t=2+3u, -8+3t=1+cu$ and $2t=3+u$
Solve the 2 equations not containing 'c'	M1		
$t=2$, $u=1$	A1		
Subst their (t,u) into equation containing c	M1		
c = −3	A1	5	
Alternative method for final 4 marks			
Solve two equations, one with 'c', for t and u in terms of c, and substitute into third equation	(M2)		
$c=-3$	(A2)		11

4.

Uses proof by induction and investigates the expression for n = 0 and n = k (must see evidence of both n = 0 and n = k being considered)	AO3.1a	M1	Let $f(n)=8^n-7n+6$ $f(0)=1+6=7$ $\Rightarrow f(n)$ is divisible by 7 when n = 0 Consider n = k Assume that f(k) is divisible by 7 $f(k+1)=8^{k+1}-7(k+1)+6$ $f(k+1)-8f(k)=56k-7(k+1)+6-48$ $f(k+1)-8f(k)=49k-49$ $f(k+1)=8f(k)+49(k-1)$ $=8f(k)+7(7k-7)$ $\therefore f(k+1)$ is divisible by 7 since $f(k)$ is divisible by 7 Therefore f(k) is divisible by 7 $\Rightarrow$ f(k + 1) is divisible by 7
Shows that statement is true for n = 0	AO1.1b	B1	
Commences argument by considering f(k+1) in terms of f(k)	AO2.1	R1	
Makes correct deduction that if f(n) is divisible by 7 then f(n+1) is also divisible by 7	AO2.2a	R1	
Completes a rigorous argument and explains how their argument proves the required result. **AG**	AO2.4	R1	Since $f(0)$ is divisible by 7 *and* f(k) is divisible by 7 $\Rightarrow$f(k + 1) is divisible by 7 then, by induction, $f(n)=8^n-7n+6$ is divisible by 7 for all integers $n \geq 0$ **AG**

5.

(i)**OD**=**OA**+**AD** or **OB**+**BC**+**CD** AEF **AD** = **BC** or **CD** = **BA** (**a** + **c** – **b**) = 2**j** + **k** (ii) **AB.CB** = |**AB**||**CB**| cos θ Scalar product of <u>any</u> 2 vectors Magnitude of <u>any</u> vector 94°(94.386…) or 1.65 (1.647…)	M1 A1 A1 **3** M1 M1 M1 A1 **4**	Connect **OD** & 2/3/4 vectors in their diag Or similar ,from their diag [i.e.if diag mislabelled, M1A1A0 possible] Or **AB.BC** i.e.scalar prod for correct pair 2 + 3 – 6 = –1 is expected √19 or 3 expected Accept 86°(85.614…) or 1.49(424..) **7**

6.

(i) For (either point) + t(difference between vectors) **M1** — 't' can be 's', 'λ' etc.

r = (3**i** + 2**j** + 3**k** or **i** + 3**j** + 4**k**)+ t(–2**i** + **j** + **k** or 2**i** – **j** – **k**) **A1** — '**r**' must be '**r**' but need not be bold

Check other formats, e.g. $ta+(1-t)b$

[2]

(ii) State/imply that their **r** and their –2**i** + **j** + **k** are perpendicular — ***M1** N.B.This *M1 is dep on M1 being earned in (i)

Consider scalar product = 0 — dep***M1**

Obtain $t=-\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$ — **A1**

Subst their t into their equation of AB — **M1**

Obtain $\frac{1}{6}$(16**i** + 13**j** + 19**k**) AEF — **A1** Accept decimals if clear

[5]